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Supersymmetric intersections of M-branes and pp-waves

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Abstract: We study supersymmetric intersections of M2 and M5 branes with different pp-waves of M-theory. We consider first M-brane probes in the background of pp-waves and determine under which conditions the embedding is supersymmetric. We particularize our formalism to the case of pp-waves with 32, 24 and 20 supersymmetries. We also construct supergravity solutions for the brane-wave system. Generically these solutions are delocalised along some directions transverse to the brane and preserve the same number of supersymmetries as in the brane probe approach.

Keywords: Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence, M-Theory.

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Contents

1. Introduction

With the advent of the AdS/CTF duality, the understanding of string theory with Ramond-Ramond backgrounds has become a subject of great interest [1]. Indeed, in order to extend the gauge theory/gravity correspondence to the regime in which the gauge coupling is small, one must quantize string theory in such backgrounds. Remarkably, there exists a background of the type IIB theory with a Ramond-Ramond flux, the maximally supersymmetric pp-wave [2], in which superstring theory is exactly solvable [3]. This supergravity solution can be obtained [4] by performing the Penrose limit of the $AdS_5 \times S^5$ geometry [5, 6]. This fact opens the possibility of studying the string theory gauge theory correspondence at the level of full string theory [7].

The study of D-branes in the pp-wave background is obviously interesting in order to have an insight on the non-perturbative phenomena of string theory in this geometry and, through the AdS/CFT correspondence, of its gauge theory dual. There are at least three ways to study D-branes in the pp-wave background. The first one is just the original Polchinski approach adapted to this case, i.e. one studies open strings with Dirichlet boundary conditions which preserve some amount of supersymmetry. This is the point of view adopted in ref. [8]. The second approach to this problem is the brane probe formalism [9], in which one considers the Dirac-Born-Infeld action for the D-brane and looks for solutions of the equation of motion such that some of the Killing spinors of the background fulfill the kappa symmetry condition. Finally, one can, as in refs. [10]–[12], try to find supergravity solutions representing the intersection between the D-brane and the pp-wave. In this case the number of supersymmetries preserved by the configuration is just the number of Killing spinors of the supergravity solution representing the pp-wave/D-brane intersection.

In eleven dimensional supergravity the maximally supersymmetric pp-wave solution was found long time ago in ref. [13]. As in the ten dimensional case, this pp-wave with 32 supersymmetries can be obtained by means of a Penrose limit of the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ solutions [4]. Actually, there exist some pp-wave backgrounds which, in addition to the 16 standard supersymmetries preserved by a generic pp-wave, are also invariant under a set of supersymmetry transformations along some so-called "supernumerary" Killing spinors [14, 15]. These backgrounds can be obtained in some cases as Penrose limits of (non-standard) brane intersections [14, 16, 17]. The matrix theory for these M-theory pp-waves was proposed in ref. [7] and is usually referred to as the BMN matrix theory. As compared with the original matrix theory, the BMN matrix action contains mass terms, both for bosons and fermions, as well as a cubic interaction term, the so-called Myers term. This action can be obtained from a matrix regularization of the supermembrane action in the pp-wave geometry [18, 19].

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On can use the BMN matrix theory to find BPS objects on the pp-wave. This is the approach followed in refs. [19]–[28]. Here we will adopt a different point of view to deal with the problem of finding supersymmetric intersections with the M-theory pp-waves. We will consider first M2 and M5 brane probes in the pp-wave background. Although our formalism is valid for a general case, we will mostly concentrate our analysis in the pp-wave backgrounds which preserve 32, 24 and 20 supersymmetries. The basic tool in the brane probe approach is kappa symmetry, which provides a condition to be satisfied by the Killing spinors if the corresponding brane embedding is to be supersymmetric.

For a given M2 and M5 brane embedding, the number of Killing spinors of the background satisfying the kappa symmetry condition is just the number of supersymmetries of the brane-wave intersection. Actually, we will restrict ourselves to branes extended along the two light-cone directions and along some fixed transverse hyperplane. The corresponding kappa symmetry matrix is just the antisymmetrized product of constant gamma matrices and the requirements of kappa symmetry reduce to a set of algebraic constraints to be satisfied by the Killing spinors. It is not difficult to perform a case by case analysis of these constraints and determine the number of supersymmetries of each possible configuration. Actually, as we will see in the explicit examples, the number of supersymmetries depends on whether the brane is located at the origin in the transverse space or at an arbitrary point. In general, some supersymmetries are lost when we move away from the origin in a generic direction. It is also interesting to determine how many supernumerary Killing spinors of the pp-wave survive in the intersection with the branes. Generically, the wave-brane intersections are not invariant under supersymmetries along supernumerary Killing spinors (specially for branes located at arbitrary points in transverse space), although there are some distinguished cases in which this does not occur.

We will also try to find supergravity solutions representing the brane-wave intersection. The natural ansatz for the metric of these solutions is obtained by including the corresponding warp factors along the directions parallel and transverse to the brane. These warp factors are powers of a single harmonic function, which depends on the coordinates transverse to the brane. In addition, we expect to have some modifications to the original quadratic profile of the pp-wave, due to the back-reaction of the brane [12]. On the other hand, the four-form field strength for these solutions is the sum of the constant flux corresponding to the brane and the standard M2 or M5 ansätze, the latter being given in terms of the derivatives of the harmonic function. Actually, in most of the cases, the equations of motion of the four-form are satisfied if the M2 or M5 are delocalised along some of their external coordinates, which implies that the harmonic function only depends on a subset of the external coordinates. Once the harmonic function is determined, the profile for the metric of the intersection can be found by integrating a second-order differential equation, which is obtained from the Einstein equations.

The analysis of the supersymmetry preserved by our supergravity solutions leads to a series of conditions for the Killing spinors, which include, in particular, the algebraic equations found in the brane probe approach. Thus, only those embeddings which preserve some supersymmetry in the brane probe approach can give rise to supersymmetric solutions of the supergravity equations. In addition, we will get some extra conditions which are a consequence of the warping of the metric and involve derivatives with respect to the external coordinates. These extra conditions fail to be satisfied by some of the brane configurations which were found to preserve some supersymmetries in the brane probe approach. In the generic situation, however, we find agreement between the number of supersymmetries obtained in the supergravity analysis and the brane probe approach for a brane located at an arbitrary transverse position.

This paper is organized as follows. In section 2 we first present the general conditions that make a brane probe supersymmetric in the pp-wave background. We then apply this general formalism to the case of the maximally supersymmetric pp-wave, which was previosly considered in [29], and to the pp-wave with 24 supersymmetries. In section 3 we introduce our ansatz for the supergravity solutions corresponding to the wave-brane intersections and discuss their supersymmetries. The values of the different components of the Ricci tensor for these metrics are given in appendix A, while the solution of a differential equation, which appears in the determination of the profile, is worked out in appendix B. In section 4 we apply our general formalism to the study of supergravity solutions corresponding to M2 and M5 branes intersecting a maximally supersymmetric pp-wave, whereas in section 5 the pp-wave with 24 supersymmetries is considered. The case of the pp-wave with 20 supersymmetries is treated in appendix C. Finally, in section 6 we summarize our results and draw some conclusions.

2. Probe analysis

Let us consider an eleven dimension pp-wave metric of the type:

$$ds^2_{11} = 2dx^+ \, dx^- + W \, (\, dx^+ \,)^2 + (\, dx^i \,)^2 \,, \tag{2.1}$$

where (x^+, x^-) are light-cone coordinates and the x^i's will be referred to as transverse coordinates. The function W is the so-called profile of the pp-wave and we will assume that only depends on the transverse coordinates x^i. On the other hand, the four-form field strength of eleven dimensional supergravity will be taken as:

$$F = dx^+ \wedge \Theta \,, \tag{2.2}$$

with Θ being:

$$\Theta = \frac{1}{6}\, \theta_{ijk} \, dx^i \wedge dx^j \wedge dx^k \,. \tag{2.3}$$

This configuration is a solution of Einstein equations if the profile W satisfies:

$$\partial_i^2 \, W = -\frac{1}{6}\, \theta_{mnl} \, \theta^{mnl} \,. \tag{2.4}$$

Any solution of the above equation gives rise to a background with 16 supersymmetries. However, for some choices of Θ and W one can have solutions with more supersymmetry. In this paper we will restrict ourselves to the case in which Θ is given by a four parameter ansatz [14, 15] of the type:[1]

$$\Theta = \mu_1 dx^1 \wedge dx^2 \wedge dx^9 + \mu_2 dx^3 \wedge dx^4 \wedge dx^9 + \mu_3 dx^5 \wedge dx^6 \wedge dx^9 + \mu_4 dx^7 \wedge dx^8 \wedge dx^9 \,, \tag{2.5}$$

[1] There also exists a seven parameter ansatz [14, 15] which, in particular, gives rise to an eleven dimensional pp-wave with 26 supersymmetries [30].

with the μ_i's being constants. It follows from eq. (2.4) that W must be a quadratic function of the transverse coordinates x^i. Actually, if we write:

$$W = -\sum_i \lambda_i^2 \, (x^i)^2 \,, \tag{2.6}$$

then, a solution of the Einstein equations which preserves at least 18 supersymmetries is obtained when the λ's and the μ's are related as [14, 15]:

$$\begin{aligned}
\lambda_1^2 &= \lambda_2^2 = \frac{1}{36} \, (\, 2\mu_1 - \mu_2 - \mu_3 - \mu_4 \,)^2 \,, \\
\lambda_3^2 &= \lambda_4^2 = \frac{1}{36} \, (\, -\mu_1 + 2\mu_2 - \mu_3 - \mu_4 \,)^2 \,, \\
\lambda_5^2 &= \lambda_6^2 = \frac{1}{36} \, (\, -\mu_1 - \mu_2 + 2\mu_3 - \mu_4 \,)^2 \,, \\
\lambda_7^2 &= \lambda_8^2 = \frac{1}{36} \, (\, -\mu_1 - \mu_2 - \mu_3 + 2\mu_4 \,)^2 \,, \\
\lambda_9^2 &= \frac{1}{9} \, (\, \mu_1 + \mu_2 + \mu_3 + \mu_4 \,)^2 \,.
\end{aligned} \tag{2.7}$$

In the study of the supersymmetry of brane probes in the above geometry we must know the Killing spinors of the background. These spinors are solutions of the equation $\nabla_M \, \epsilon = \Omega_M \, \epsilon$, where ∇_M is the covariant derivative and Ω_M is given by:

$$\Omega_M = \frac{1}{288} \, F_{PQRS} \left(\Gamma^{PQRS}{}_{M} + 8 \, \Gamma^{PQR} \, \delta^S_M \right) . \tag{2.8}$$

In order to write the general form of the Killing spinors, let us define the matrix:

$$\theta = \frac{1}{6} \, \theta_{\hat{i}\,\hat{j}\,\hat{k}} \;\; \Gamma^{\hat{i}\,\hat{j}\,\hat{k}} \,. \tag{2.9}$$

In eq. (2.9), and in what follows, hatted indices denote flat components with respect to the basis of one-forms given in appendix A (see eq. (A.2)). Then [31], the Killing spinors ϵ take the form:

$$\epsilon = (\, 1 + x^i \, \Omega_i) \, e^{x^+ \, \Omega_+} \, \chi \,, \tag{2.10}$$

where χ is a constant spinor and the dependence of ϵ on the coordinates x^+ and x^i is determined by the action of the matrices Ω_+ and Ω_i on χ. These matrices are given in terms of θ as follows:

$$\Omega_+ = -\frac{1}{12} \, \theta \, \left[\, \Gamma_{\hat{-}} \Gamma_{\hat{+}} + 1 \, \right] , \qquad \Omega_i = \frac{1}{24} \, \left[\, 3\theta \Gamma_{\hat{i}} + \Gamma_{\hat{i}} \theta \, \right] \, \Gamma_{\hat{-}} \,. \tag{2.11}$$

The spinor χ is, in general, not arbitrary but determined by some algebraic constraints. In particular, there are always 16 Killing spinors, obtained by solving the equation $\Gamma_{\hat{-}} \, \chi = 0$, which are the so-called standard spinors. Notice that these standard Killing spinors do not depend on the transverse coordinates x^i and they can only depend on the light-cone coordinate x^+. The spinors ϵ for which $\Gamma_{\hat{-}} \, \chi \neq 0$ are called supernumerary Killing spinors. For the background we have written above there are at least two of them. These supernumerary spinors have a nontrivial dependence on the transverse coordinates x^i.

Let us now place an M-brane probe in the above pp-wave background. The supersymmetry preserved by the probe is determined by the solutions of the equation:

$$\Gamma_{\kappa}\,\epsilon = \epsilon\,, \tag{2.12}$$

where Γ_κ is the so-called kappa symmetry matrix of the brane probe, ϵ is a Killing spinor of the background and it should be understood that both sides of this equation are evaluated on the worldvolume of the brane.

The kappa symmetry matrix for an M2-brane is:

$$\Gamma_{\kappa}^{M2} = \frac{1}{3!\sqrt{-\det g}}\,\epsilon^{\mu\nu\rho}\,\gamma_{\mu\nu\rho}\,, \tag{2.13}$$

where g is the determinant of the induced metric, $\gamma_\mu = \partial_\mu x^M\,E_M^{\widehat{P}}\,\Gamma_{\widehat{P}}$ are the induced Γ-matrices, with $E_M^{\widehat{P}}$ being the vierbeins of the eleven dimensional metric G_{MN}, defined as:

$$G_{MN} = E_M^{\widehat{P}}\,E_N^{\widehat{Q}}\,\eta_{\widehat{P}\widehat{Q}}\,, \tag{2.14}$$

where the flat metric $\eta_{\widehat{P}\widehat{Q}}$ is such that $\eta_{\hat{+}\hat{-}} = 1$. The values of these vierbeins are:

$$\begin{aligned} &E_{-}^{\hat{-}} = 1\,, \qquad E_{+}^{\hat{-}} = \frac{W}{2}\,, \\ &E_{+}^{\hat{+}} = 1\,, \qquad E_{-}^{\hat{+}} = 0\,, \qquad\qquad E_i^{\hat{j}} = \delta_{ij}\,. \end{aligned} \tag{2.15}$$

Embedding the M2-brane in such a way that the worldvolume coordinates are $\xi^\mu = (\,x^+, x^-, x^a\,)$ with the other coordinates being constant, we get:

$$\Gamma_{\kappa}^{M2} = \Gamma_{\hat{-}\,\hat{+}\hat{a}}\,. \tag{2.16}$$

Let us next consider an M5-brane probe in the so-called PST formalism [32]. We will take the worldvolume 3-form H of this approach equal to zero. If a is the PST scalar [32], the kappa symmetry matrix is:

$$\Gamma_{\kappa}^{M5} = \frac{1}{5!\,\sqrt{-\det g}}\,\frac{1}{(\,\partial\cdot a)^2}\,\partial_m\,a\,\gamma^m\,\gamma_{i_1\cdots i_5}\,\epsilon^{i_1\cdots i_5 n}\,\partial_n\,a\,. \tag{2.17}$$

We will embed the M5-brane in such a way that the worldvolume coordinates are $\xi^\mu = (\,x^+, x^-, x^{a_1}, \cdots, x^{a_4}\,)$, with the other x^i's constant. The field a can be gauge-fixed to some convenient value [32]. Let us take it to be $a = x^{a_4}$, i.e. a is equal to the "last" worldvolume spatial coordinate. Then, the kappa symmetry matrix (2.17) takes the form:

$$\Gamma_{\kappa}^{M5} = \Gamma_{\hat{-}\,\hat{+}\hat{a}_1\cdots\hat{a}_4}\,. \tag{2.18}$$

The M-branes can be extended along only one of the light cone coordinates. Notice that it cannot be extended only along the x^- coordinate since, as the x^-x^- component of the metric is zero, the worldvolume metric would be degenerate (with vanishing determinant) and the corresponding configuration is not admissible. Therefore, only M-branes extended along x^+ and other two or five transverse coordinates are, in principle, possible. The induced matrix along the light cone coordinate for such a configuration is:

$$\gamma_+ = \Gamma_{\hat{+}} + \frac{W}{2}\Gamma_{\hat{-}}\,. \tag{2.19}$$

Notice the dependence of γ_+ on the transverse coordinates. This dependence is transmitted to the kappa symmetry matrix. For example, for a M2-brane extended along the coordinates (x^+, x^{a_1}, x^{a_2}), the matrix Γ_κ is:

$$\Gamma_\kappa = \frac{1}{\sqrt{-W}}\left[\Gamma_{\hat{+}} + \frac{W}{2}\Gamma_{\hat{-}}\right]\,\Gamma_{\hat{x}^{a_1}\,\hat{x}^{a_2}}\,. \tag{2.20}$$

Due to this extra coordinate dependence it is impossible to realize the kappa symmetry condition $\Gamma_\kappa\epsilon = \epsilon$. The same happens for an M5-brane embedding of this type. Therefore, in what follows, we would only consider M2- and M5-branes extended along the two light-cone coordinates. The corresponding kappa symmetry matrices will be given by the constant matrices written in eqs. (2.16) and (2.18) respectively.

Let us rewrite the Killing spinors (2.10) as:

$$\epsilon = e^{x^i\Omega_i}\,\chi^{(+)}\,, \tag{2.21}$$

where we have taken into account that $\Omega_i\Omega_j = 0$ and $\chi^{(+)}$ is given by:

$$\chi^{(+)} = e^{x^+\Omega_+}\,\chi\,. \tag{2.22}$$

In terms of $\chi^{(+)}$ the condition $\Gamma_\kappa\epsilon = \epsilon$ can be written as:

$$e^{-x^i\Omega_i}\,\Gamma_\kappa\,e^{x^i\Omega_i}\,\chi^{(+)} = \chi^{(+)}\,. \tag{2.23}$$

Expanding the exponentials on the right-hand side of (2.23), and comparing the dependence on the coordinates x^i of both sides of the equation, we get:

$$\Gamma_\kappa\,\chi^{(+)} = \chi^{(+)}\,, \qquad [\,\Gamma_\kappa\,,\,\Omega_i\,]\,\chi^{(+)} = 0\,, \qquad \Omega_i\,\Gamma_\kappa\,\Omega_i\,\chi^{(+)} = 0\,. \tag{2.24}$$

The last condition in (2.24) is automatic for the type of embeddings we are considering, since $\Gamma^2_{\hat{-}} = 0$. Let us consider the first two conditions. Taking $x^+ = 0$ on these equations we get the following algebraic conditions on the constant spinor χ:

$$\Gamma_\kappa\,\chi = \chi\,, \qquad [\,\Gamma_\kappa\,,\,\Omega_i\,]\,\chi = 0\,. \tag{2.25}$$

Moreover, taking into account that $\Gamma_\kappa^2 = 1$ and the first equation in (2.25), one easily proves that $\Gamma_\kappa\,\chi^{(+)} = \chi^{(+)}$ is equivalent to

$$e^{x^+\Gamma_\kappa\Omega_+\Gamma_\kappa}\,\chi = e^{x^+\Omega_+}\,\chi\,, \tag{2.26}$$

which, in turn, is satisfied if and only if:

$$[\,\Gamma_\kappa\,,\,\Omega_+\,]\,\chi = 0\,. \tag{2.27}$$

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Similarly, one can prove that $[\,[\Gamma_\kappa\,,\,\Omega_i\,]\,,\Omega_+\,]\,\chi\,=\,0$ and, after taking into account that $[\,\Gamma_\kappa\,,\,\Omega_i\,]\,\chi = 0$, one concludes that we must have:

$$[\,\Gamma_\kappa\,,\,\Omega_i\,]\,\Omega_+\,\chi = 0\,. \tag{2.28}$$

The algebraic equations (2.25), (2.27) and (2.28) for the constant spinor χ are equivalent to the kappa symmetry condition $\Gamma_\kappa\,\epsilon = \epsilon$ and will be the starting point of our analysis of the supersymmetry preserved by the different brane probe configurations. First of all, notice that, from the expression of Ω_+ and the fact that Γ_κ always commutes with $\Gamma_{\hat{-}}\Gamma_{\hat{+}}$, eq. (2.27) can be written as:

$$[\,\Gamma_\kappa\,,\,\theta\,]\,\big(\Gamma_{\hat{-}}\Gamma_{\hat{+}}+1\big)\,\chi = 0\,. \tag{2.29}$$

Moreover, the matrix $\Gamma_{\hat{-}}\Gamma_{\hat{+}}+1$ has no non-trivial zero modes. Indeed, if χ is such a zero mode, it would satisfy $\Gamma_{\hat{-}}\Gamma_{\hat{+}}\,\chi = -\chi$. By multiplying this last equation by $\Gamma_{\hat{+}}$, and using the anticommutation relation $\{\Gamma_{\hat{+}},\Gamma_{\hat{-}}\} = 2$, one obtains $\Gamma_{\hat{+}}\,\chi = 0$. Plugging this result in the zero-mode equation one gets that $\chi = 0$. Actually, the matrix $\Gamma_{\hat{-}}\Gamma_{\hat{+}}+1$ is invertible and its inverse is $(\Gamma_{\hat{+}}\Gamma_{\hat{-}}+1)/3$. It follows that one must have:

$$[\,\Gamma_\kappa\,,\,\theta\,]\,\chi = 0\,. \tag{2.30}$$

Following the same steps we can also prove that eq. (2.28) is equivalent to:

$$[\,\Gamma_\kappa\,,\,\Omega_i\,]\;\theta\,\chi = 0\,. \tag{2.31}$$

Notice that to satisfy eq. (2.30) either Γ_κ and θ commute or else χ is a zero mode of $[\,\Gamma_\kappa\,,\,\theta\,]$. To study the appearance of such zero modes, let us split θ in two pieces, $\theta = \theta\,' + \theta\,''$, such that:

$$\{\Gamma_\kappa\,,\,\theta\,'\} = 0\,, \qquad [\Gamma_\kappa\,,\,\theta\,''] = 0\,. \tag{2.32}$$

Then, it is clear that $[\Gamma_\kappa\,,\,\theta\,] = -2\theta\,'\Gamma_\kappa$ and eq. (2.30) implies that χ must be a zero mode of $\theta\,'$. Let us similarly split the Ω_i's as $\Omega_i = \Omega'_i + \Omega''_i$, where Ω'_i (Ω''_i) is given by the second expression in (2.11) with θ substituted by $\theta\,'(\theta\,'')$. In order to study the algebraic conditions involving the Ω_i's, it is important to distinguish between coordinates along the worldvolume of the brane an those orthogonal to it. Accordingly, let us split the x^i 's as $x^i = (x^a, x^\alpha)$, where x^a are the coordinates along which the M-brane is extended and the x^α's are constant and determine the location of the brane in the transverse space. It is important to point out that, when the brane is placed at $x^\alpha = 0$, we should consider only the conditions involving the Ω_a's. Moreover, since $[\,\Gamma_\kappa\,,\,\Gamma_{\hat{-}\hat{a}}\,] = \{\,\Gamma_\kappa\,,\,\Gamma_{\hat{-}\hat{\alpha}}\,\} = 0$, one has:

$$\begin{aligned}
&[\Gamma_\kappa\,,\,\Omega'_a\,] = -2\Omega_a{}'\,\Gamma_\kappa\,, &\qquad& [\Gamma_\kappa\,,\,\Omega''_a\,] = 0\\
&[\Gamma_\kappa\,,\,\Omega'_a\,] = 0\,, && [\Gamma_\kappa\,,\,\Omega''_\alpha\,] = -2\Omega''_\alpha\,\Gamma_\kappa\,.
\end{aligned} \tag{2.33}$$

It is now straightforward to reduce the conditions (2.25), (2.30) and (2.31) to the following set of equations:

$$\boxed{\begin{array}{ll}
\Gamma_\kappa\,\chi = \chi & \theta\,'\,\chi = 0\\[4pt]
\Omega'_a\,\chi = 0 & \Omega'_a\,\theta''\,\chi = 0\\[4pt]
\Omega''_\alpha\,\chi = 0 & \Omega''_\alpha\theta''\,\chi = 0.
\end{array}} \tag{2.34}$$

In particular, when Γ_κ commutes (anticommutes) with θ (i.e. when $\theta'(\theta'')$ vanishes) the conditions involving Ω_a (Ω_α) are absent and the system (2.34) collapses to one of the following two lines, in addition to the equation $\Gamma_\kappa\chi = \chi$:

$$\begin{aligned} [\,\Gamma_\kappa\,,\,\theta\,] = 0 &\Longrightarrow \Omega_\alpha\,\chi\, = 0\,, \qquad \Omega_a\,\theta\,\chi\, = 0 \\ \{\,\Gamma_\kappa\,,\,\theta\,\} = 0 &\Longrightarrow \quad \theta\,\chi\, = 0\,, \qquad \Omega_a\,\chi\, = 0\,. \end{aligned} \tag{2.35}$$

The analysis of eqs. (2.34) for the different brane embeddings will allow us to determine their supersymmetry. Actually, for the pp-wave backgrounds studied in the main text, the simplified equations (2.35) will be enough and we will be able to identify easily those configurations which preserve some amount of supersymmetry.

2.1 Maximally supersymmetric pp-wave

The metric of the maximally supersymmetric pp-wave background in M-theory is [13]:

$$ds^2_{11} = 2dx^+\,dx^- - \left[\left(\frac{\mu}{3}\right)^2\,\vec{y}^{\,2} + \left(\frac{\mu}{6}\right)^2\,\vec{z}^{\,2}\right]\,(\,dx^+\,)^2 + d\vec{y}^{\,2} + d\vec{z}^{\,2}\,, \tag{2.36}$$

where μ is a scale, $\vec{y} = (\,y^1\,,\,y^2\,,\,y^3\,)$ and $\vec{z} = (\,z^1\,,\cdots,\,z^6\,)$. We have labeled the transverse coordinates x^i as $x^i = y^i$ for $i = 1,\cdots 3$ and $x^{3+j} = z^j$ for $j = 1,\cdots 6$. The four-form F takes the value:

$$F = \mu\,dx^+ \wedge dy^1 \wedge dy^2 \wedge dy^3\,. \tag{2.37}$$

This background can be obtained by taking $\mu_1 = \mu$ and $\mu_2 = \mu_3 = \mu_4 = 0$ in our four parameter ansatz of eqs. (2.5)–(2.7). Notice that the matrix θ, defined in eq. (2.9), is now given by:

$$\theta = \mu\,\Gamma_{\hat{y}^1\,\hat{y}^2\,\hat{y}^3} \equiv \mu\,I\,, \tag{2.38}$$

where we have defined the matrix I. Moreover, by using the value of θ given in eq. (2.38) in the definition of the Ω_i's (eq. (2.11)), one easily obtains their expressions, namely:

$$\Omega_{y^i} = \frac{\mu}{6}\,I\,\Gamma_{\hat{y}^i}\,\Gamma_{\hat{-}}\,, \qquad \Omega_{z^j} = \frac{\mu}{12}\,I\,\Gamma_{\hat{z}^j}\,\Gamma_{\hat{-}}\,. \tag{2.39}$$

The Killing spinors for this supergravity solution are given by the general expression (2.10), where χ is an arbitrary constant spinor. Therefore, it has 32 supersymmetries and, actually, it can be obtained by performing the Penrose limit of the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ solutions [4].

The SUSY properties of the brane probes will depend of the spatial directions occupied by the branes on the 3+6 split. We shall consider test M2 and M5 branes in this background extended along the directions $+$, $-$, m coordinates y^a and n coordinates z^b. We shall denote these configurations as $(+,-,m,n)$ branes. Clearly $m+n = 1$ for a M2-brane, whereas $m+n = 4$ for a M5-brane. Thus, the configurations to explore of the $(+,-,m,n)$ type for the M2-brane are:

$$(+,-,1,0)\,, \qquad (+,-,0,1)\,. \tag{2.40}$$

For the M5-brane we have the following possibilities of the $(+,-,m,n)$ type:

$$\begin{aligned} &(+,-,3,1)\,, \qquad (+,-,2,2)\,, \\ &(+,-,1,3)\,, \qquad (+,-,0,4)\,. \end{aligned} \tag{2.41}$$

For such a $(+,-,m,n)$ brane configuration we will take the following set of worldvolume coordinates:

$$\xi^i = (\, x^+, x^-, y^{a_1}, \dots, y^{a_m}, z^{b_1}, \dots, z^{b_n}\,)\,, \tag{2.42}$$

while the other y's and z's are transverse constant scalars. First of all we will consider all possible configurations with all these scalars equal to zero and, afterwards, we shall explore the possibility of giving them a non vanishing value.

We shall apply in our analysis the methodology which we have developed for the general case. First of all, we consider the possible ways of fulfilling eq. (2.30). A kappa symmetry matrix Γ_κ of the types written in eqs. (2.16) and (2.18) either commutes or anticommutes with the matrix θ of eq. (2.38). Notice that θ has no zero modes, since its eigenvalues are $\pm\mu$. Thus, if $\{\Gamma_\kappa, \theta\} = 0$ eq. (2.30) has no solution, and the only possibility left is that $[\,\Gamma_\kappa, \theta\,] = 0$. Then, according to eq. (2.35), these configurations without transverse scalars are 1/2 supersymmetric, with 16 supersymmetries, which correspond to the spinors satisfying $\Gamma_\kappa\chi = \chi$.

If the brane probe is placed at a non-zero value of the transverse coordinates y^α and z^α, we must study the zero modes of Ω_{y^α} and Ω_{z^α} (see eq. (2.35)). By inspecting the form of these matrices in eq. (2.39), one readily realizes that χ is a zero mode of them if and only if $\Gamma_{\hat{-}}\,\chi = 0$, which means that the corresponding Killing spinors are all standard. Thus, we are led to introduce a second projection on χ and, as a consequence, the configuration preserves 8 supersymmetries, i.e. is 1/4 supersymmetric.

At this point it is interesting to notice that, as $\{\,\Gamma_{\hat{+}}, \Gamma_{\hat{-}}\,\} = 2$, any spinor χ can be decomposed as $\chi = \chi_+ + \chi_-$ with:

$$\chi_\pm = \frac{1}{2}\,\Gamma_{\hat{\pm}}\,\Gamma_{\hat{\mp}}\,\chi \;\equiv\; \mathcal{P}_\pm\,\chi\,. \tag{2.43}$$

The operators $\mathcal{P}_\pm \;\equiv\; \frac{1}{2}\,\Gamma_{\hat{\pm}}\,\Gamma_{\hat{\mp}}$ are clearly projectors, since $\mathcal{P}_+ + \mathcal{P}_- = 1$, $\mathcal{P}_+\mathcal{P}_- = \mathcal{P}_-\mathcal{P}_+ = 0$ and $(\mathcal{P}_\pm)^2 = \mathcal{P}_\pm$. As $\Gamma_{\hat{\pm}}\,\chi_\pm = 0$, it is clear that $\mathcal{P}_\pm$ projects on the subspace of spinors such that $\Gamma_{\hat{\pm}}\,\chi_\pm = 0$. Actually, by multiplying the condition $\Gamma_{\hat{\pm}}\,\chi_\pm = 0$ by $\Gamma_{\hat{\mp}}$ and using that $\Gamma_{\hat{\mp}}\Gamma_{\hat{\pm}} = -\Gamma_{\hat{\pm}}\Gamma_{\hat{\mp}} + 2$, one gets $\Gamma_{\hat{\pm}}\Gamma_{\hat{\mp}}\,\chi_\pm \;=\; 2\chi_\pm$, i.e. the condition $\Gamma_{\hat{\pm}}\,\chi_\pm = 0$ is equivalent to $\mathcal{P}_\pm\chi_\pm = \chi_\pm$. On the other hand, it is interesting to notice that the condition $\mathcal{P}_\pm\chi_\pm = \chi_\pm$ is equivalent to $\Gamma_{\hat{\pm}\hat{\mp}}\,\chi_\pm = \chi_\pm$.

It follows from the above discussion that when $[\,\Gamma_\kappa\,, \theta] \;=\; 0$ and the brane probe is placed at an arbitrary point in transverse space, the 8 supersymmetries of the system are characterized by spinors χ which satisfy:

$$\Gamma_\kappa\,\chi = \mathcal{P}_-\chi = \chi\,. \tag{2.44}$$

As a consistency check of the projection (2.44), one easily verifies that $[\Gamma_\kappa, \mathcal{P}_-] = 0$.

In conclusion, we have to analyze in each case whether or not Γ_κ commutes with the matrix θ of eq. (2.38). We will do it separately for M2 and M5 branes in the next subsections. The same results for the probe analysis in this maximally supersymmetric pp-wave have been found in ref. [29].

M2	# Susys without scalars	# Susys with scalars
$(+,-,1,0)$	16	8
M5		
$(+,-,2,2)$	16	8
$(+,-,0,4)$	16	8

Table 1: Supersymmetric intersections of M2 and M5 branes with the maximally SUSY pp-wave background.

2.1.1 M2-brane configurations

Let us study the supersymmetry preserved by a $(+,-,1,0)$ configuration. Without loss of generality we can assume that the M2-brane is extended along the direction y^1. The corresponding Γ_κ matrix is:

$$\Gamma_\kappa^{M2} = \Gamma_{\widehat{-}\widehat{+}}\Gamma_{\widehat{y}^1}\,. \tag{2.45}$$

It is straightforward to verify that the matrix Γ_κ displayed in eq. (2.45) commutes with I and, thus, with θ. Therefore, according to our general analysis, this configuration preserves 16 supersymmetries when the probe is located at the origin of the transverse space and 8 supersymmetries when the M2-brane is placed at an arbitrary point.

It is also immediate to check that the kappa symmetry matrix of the remaining $(+,-,0,1)$ M2-brane configuration does not commute with θ and, as a consequence, is non-supersymmetric.

2.1.2 M5-brane configurations

For a $(+,-,m,n)$ configuration the kappa symmetry matrix for the M5-brane is:

$$\Gamma_\kappa^{M5} = \Gamma_{\widehat{-}\widehat{+}}\,\Gamma_{\widehat{y}^{a_1}\ldots\widehat{y}^{a_m}}\,\Gamma_{\widehat{z}^{b_1}\ldots\widehat{z}^{b_n}}\,. \tag{2.46}$$

After some calculation one can check that:

$$\Gamma_\kappa\, I = (-1)^n\, I\,\Gamma_\kappa\,, \tag{2.47}$$

and, thus, Γ_κ commutes with θ only if $n \in 2\mathbb{Z}$ (or if $m \in 2\mathbb{Z}$ since $n+m$ is even). Therefore we conclude that the configurations with n even and located at the origin of the transverse space are 1/2 supersymmetric. They are:

$$(+,-,2,2)\,, \qquad (+,-,0,4)\,, \tag{2.48}$$

while these same embeddings with excited transverse scalars are only 1/4 supersymmetric.

In the table 1 we summarize our results for M-branes in the maximally supersymmetric pp-wave background. We include only the configurations which preserve some amount of supersymmetry.

2.2 pp-Wave with 24 supersymmetries

Let us now split the coordinates of the eleven dimensional spacetime as: $x^\mu = (x^+, x^-, x^i) = (x^+, x^-, \vec{y}, \vec{z}, x^9)$, where $\vec{y}$ and $\vec{z}$ are four component vectors, i.e., $\vec{y} = (y^1, y^2, y^3, y^4)$ and $\vec{z} = (z^1, z^2, z^3, z^4)$. Clearly $x^i = y^i$ and $x^{4+i} = z^i$ for $i = 1, \cdots, 4$. The metric of the pp wave with 24 supersymmetries is:

$$ds^2_{11} = 2dx^+ dx^- - \frac{\mu^2}{4} \vec{y}^{\,2} \, (dx^+)^2 + d\vec{y}^{\,2} + d\vec{z}^{\,2} + (dx^9)^2 , \tag{2.49}$$

and the four-form F is:

$$F = \mu \, \left[dx^+ \wedge dy^1 \wedge dy^2 \wedge dx^9 + dx^+ \wedge dy^3 \wedge dy^4 \wedge dx^9 \right] . \tag{2.50}$$

This supergravity solution can be obtained from the general four parameter ansatz of eqs. (2.5)–(2.7) by taking $\mu_1 = -\mu_2 = \mu$, $\mu_3 = -\mu_4 = 0$, after a suitable relabeling of the transverse coordinates. In order to characterize the supersymmetry of this background, let us introduce the following matrix:

$$J \equiv \Gamma_{\widehat{y}^1} \Gamma_{\widehat{y}^2} \Gamma_{\widehat{x}^9} + \Gamma_{\widehat{y}^3} \Gamma_{\widehat{y}^4} \Gamma_{\widehat{x}^9} , \tag{2.51}$$

in terms of which θ is simply:

$$\theta = \mu \, J , \tag{2.52}$$

and the matrices Ω_i are given by:

$$\Omega_{y^i} = \frac{\mu}{24} \left[3J\Gamma_{\widehat{y}^i} + \Gamma_{\widehat{y}^i} J \right] \Gamma_{\widehat{-}} , \qquad \Omega_{z^j} = \frac{\mu}{12} J \Gamma_{\widehat{z}^j} \Gamma_{\widehat{-}} , \qquad \Omega_{x^9} = \frac{\mu}{6} J \Gamma_{\widehat{x}^9} \Gamma_{\widehat{-}} , \tag{2.53}$$

The standard Killing spinors are, in this case, 16 spinors of the form:

$$\epsilon^{st} = e^{-\frac{\mu}{4} x^+ J} \, \chi^{st} , \qquad \Gamma_{\widehat{-}} \chi^{st} = 0 . \tag{2.54}$$

This background has, in addition, 8 supernumerary Killing spinors. They are of the form:

$$\epsilon^{sn} = \left(1 + \frac{\mu}{8} \Gamma_{-} J \sum_{i=1}^{4} y^i \, \Gamma_{y^i} \right) \chi^{sn} , \tag{2.55}$$

where χ^{sn} is a constant spinor such that $\Gamma_{\widehat{-}} \chi^{sn} \neq 0$ and which satisfies the condition:

$$J \, \chi^{sn} = 0 . \tag{2.56}$$

Notice that the supernumerary spinors are all independent of x^+. On the contrary the standard Killing spinors depend on x^+ except when $J\chi^{sn} = 0$. Thus, in this case we have 8 standard Killing spinors independent of x^+. Moreover, if we define the matrix $\Gamma^{(y)}$ as:

$$\Gamma^{(y)} \equiv \Gamma_{\widehat{y}^1 \widehat{y}^2 \widehat{y}^3 \widehat{y}^4} , \tag{2.57}$$

one can immediately prove that:

$$J\chi = 0 \qquad \Leftrightarrow \qquad \Gamma^{(y)} \, \chi = \chi . \tag{2.58}$$

Let us consider an M-brane probe in the previous background. Notice that the 9 transverse dimensions are split as 4+4+1. Actually, the four $\vec{y}$ coordinates are not equivalent and, in general, it is important to distinguish between the two sets of y^i coordinates. We shall denote by $(+,-,(m_1,m_2),n,p)$ to a M-brane embedding along m_1 coordinates of the set $\{y^1,y^2\}$ and m_2 coordinates of the set $\{y^3,y^4\}$, with n and p being the number of z^i and x^9 coordinates respectively. Obviously $m_1+m_2+n+p=1$ for an M2-brane and $m_1+m_2+n+p=4$ for an M5-brane. Moreover, we have the following equivalence relation:

$$(+,-,(m_1,m_2),n,p) \approx (+,-,(m_2,m_1),n,p)\,. \tag{2.59}$$

In order to study the number of supersymmetries of the background preserved by the probe, let us come back to our general formalism and, in particular, to eq. (2.30). An important remark concerning this equation is that now θ has zero modes. Indeed $\theta\chi=0$ iff $J\chi=0$. Thus, according to eq. (2.58), it is possible to solve eq. (2.30) when $\{\Gamma_\kappa\,,\,\theta\}=0$, provided we require that $\Gamma^{(y)}\,\chi=\chi$. On the contrary, if Γ_κ commutes with one term in J and anticommutes with the other, it is impossible to find a spinor χ satisfying (2.30). Indeed, in this case $[\Gamma_\kappa\,,\,\theta]=-2\theta'\Gamma_\kappa$ reduces to a single antisymmetrized product of Γ-matrices, which has no zero modes.

We are thus led to consider the two possible situations of eq. (2.35). Notice that, when $[\Gamma_\kappa\,,\,\theta]=0$ and the brane is located at the origin of coordinates, the only additional condition required to χ is $\Gamma_\kappa\,\chi=\chi$ and, therefore, the number of supersymmetries in this case is 1/2 of that of the background, i.e. 12. On the other hand, if $\{\Gamma_\kappa\,,\,\theta\}=0$, the conditions in (2.35) do not involve the Ω_α's and, thus, the number of supersymmetries does not change when we move the brane away from the origin.

Another interesting observation to study the fulfillment of eq. (2.35) is the fact that the matrices Ω_{y^i} cannot have supernumerary zero modes. Indeed, a supernumerary zero mode χ^{sn} of, say, Ω_{y^1} must be a zero mode of $3J\Gamma_{\hat{y}^1}+\Gamma_{\hat{y}^1}J$. After taking into account the explicit expression of J (eq. (2.51)), one immediately realizes that such a χ^{sn} must also be a zero mode of $2\Gamma_{\hat{y}^2}-\Gamma_{\hat{y}^1}\Gamma_{\hat{y}^3}\Gamma_{\hat{y}^4}$, which is impossible. Notice that this argument does not apply to the matrices Ω_{z^j} and Ω_{x^9}, whose supernumerary zero modes must satisfy the condition written in eq. (2.58).

As in the previously studied case, we shall analyze separately the M2 and M5 cases.

2.2.1 M2-brane configurations

Although the four $\vec{y}$ coordinates are not equivalent, since there is only one transverse coordinate along the M2-brane worldvolume, and due to the equivalence (2.59), the distinction among the $\vec{y}$ coordinates is irrelevant in this M2 case. Accordingly, we shall denote by $(+,-,m,n,p)$ to a M2-brane configuration extended along m coordinates y, n coordinates z and p coordinates x^9 $(m,n,p=0,1)$. It is obvious that we have the following possibilities:

$$(+,-,1,0,0)\,,\qquad (+,-,0,1,0)\,,\qquad (+,-,0,0,1)\,. \tag{2.60}$$

The kappa symmetry matrix Γ_κ^{M2} was written in general in eq. (2.16). A simple calculation yields the result that Γ_κ^{M2} and J only commute for the $(+,-,0,0,1)$ configuration,

Configuration	# SUSYs without scalars	# SUSYs with scalars
$(+,-,0,1,0)$	8 (4+4)	8 (4+4)
$(+,-,0,0,1)$	12 (8+4)	8 (8+0)

Table 2: Supersymmetric intersections of M2 branes with the 3/4 SUSY pp-wave background.

whereas they anticommute for the $(+,-,0,1,0)$ embedding. Finally, in the $(+,-,1,0,0)$ case, Γ_κ^{M2} commutes with one term in J and anticommutes with the other and, therefore, this embedding does not preserve any supersymmetry.

Let us first consider in detail the $(+,-,0,1,0)$ system. According to eq. (2.35), the spinor χ must satisfy that $J\chi = 0$ and $\Omega_{z^a}\chi = 0$. For standard spinors the second condition is automatic, while the first one requires the introduction of a new projection. Thus, the preserved standard spinors χ^{st} are four constant spinors satisfying the conditions $\mathcal{P}_-\,\chi^{st} = \Gamma_\kappa^{M2}\,\chi^{st} = \Gamma^{(y)}\,\chi^{st} = \chi^{st}$. Notice that these three projections commute among themselves, as it should. Moreover, from the form of Ω_{z^a} as given in eq. (2.53), we learn that the conditions of eq. (2.35) are already satisfied by the supernumerary spinors of the background and we only have to impose the condition $\Gamma_\kappa^{M2}\,\chi^{sn} = \chi^{sn}$, which gives four spinors of this type. In general, we will say that a configuration is $A(B+C)$ supersymmetric if it preserves A supersymmetries, being $B(C)$ the number of them corresponding to standard (supernumerary) spinors (obviously $A = B + C$). With this notation the $(+,-,0,1,0)$ system is $8(4+4)$ supersymmetric. Notice that, according to eq. (2.35), this configuration preserves the same number of supersymmetries at any point in transverse space.

Let us now consider the $(+,-,0,0,1)$ configuration. As Γ_κ^{M2} commutes with θ in this case, this embedding is $12(8+4)$ supersymmetric when the brane is located at the origin. The corresponding spinors are the original ones is eqs. (2.54) and (2.55) with the extra projections $\Gamma_\kappa^{M2}\chi^{st} = \chi^{st}$ and $\Gamma_\kappa^{M2}\chi^{sn} = \chi^{sn}$. If constant scalars are excited we have to impose the condition $\Omega_{y^\alpha}\chi = 0$, which is impossible to satisfy for supernumerary spinors. Therefore, the supernumerary spinors are lost away from the origin and we are left with a $8(8+0)$ supersymmetric system.[2]

The situation for M2-branes is summarized in the table 2.

We have only included the configurations with some supersymmetry and we have explicitly indicated the number of standard and supernumerary supersymmetries.

2.2.2 M5-brane configurations

As in eq. (2.18), the kappa symmetry matrix Γ_κ^{M5} will be taken as the antisymmetrized product of the Dirac matrices along the worldvolume directions. In order to analyze the supercharges associated to standard Killing spinors which are preserved, we have to char-

[2]We could move the brane away from the origin in the z-directions only. In this case the four supernumerary supersymmetries are still preserved. However, in what follows for a configuration with scalars excited we will mean the case in which the brane is located at a generic point in transverse space.

Configuration	# SUSYs without scalars	# SUSYs with scalars
$(+,-,(2,2),0,0)$	12 (8+4)	12 (8+4)
$(+,-,(2,0),2,0)$	12 (8+4)	8 (8+0)
$(+,-,(0,0),4,0)$	12 (8+4)	8 (8+0)
$(+,-,(1,1),1,1)$	12 (8+4)	8 (8+0)
$(+,-,(1,1),2,0)$	4 (4+0)	4 (4+0)
$(+,-,(2,0),1,1)$	4 (4+0)	4 (4+0)
$(+,-,(0,0),3,1)$	8 (4+4)	8 (4+4)

Table 3: Supersymmetric intersections of M5 branes with the 3/4 SUSY pp-wave background.

acterize those configurations for which Γ_κ^{M5} commutes or anticommutes with J. It can be proved that $[\Gamma_\kappa^{M5}, J] = 0$ for:

$$\begin{aligned}
&(+,-,(2,2),0,0)\,,\\
&(+,-,(2,0),2,0) \approx (+,-,(0,2),2,0)\,,\\
&(+,-,(0,0),4,0)\,,\\
&(+,-,(1,1),1,1)\,.
\end{aligned} \tag{2.61}$$

Moreover, $\{\Gamma_\kappa^{M5}, J\} = 0$ for:

$$\begin{aligned}
&(+,-,(1,1),2,0)\,,\\
&(+,-,(2,0),1,1) \approx (+,-,(0,2),1,1)\,,\\
&(+,-,(0,0),3,1)\,.
\end{aligned} \tag{2.62}$$

Notice $[\Gamma_\kappa, \Gamma^{(y)}] = 0$ if the number of y^i coordinates in the worldvolume is even. This condition, which holds for all the configurations written above in (2.61) and (2.62), is needed to ensure the compatibility between the kappa symmetry projection and the one corresponding to supernumerary spinors.

The configurations (2.61) preserve 12(8+4) supersymmetries when placed at the origin ($\tilde{x}^i = 0$). Outside the origin they generically lose the supersymmetries associated to χ^{sn}, since when moving in the y-directions one is forced to impose that $\Omega_y \chi^{sn} = 0$. The exception to this behavior is the $(+,-,(2,2),0,0)$ configuration, because it has no external y-directions (of course if we displace the M5 along an external z or x^9 directions these configurations are still $12(8+4)$ supersymmetric).

The supersymmetry of the embeddings listed in (2.62) is not changed by translations in the transverse space. They all have four standard spinors which correspond to the projections $\Gamma_\kappa \chi^{st} = \mathcal{P}_- \chi^{st} = \Gamma^{(y)} \chi^{st} = \chi^{st}$ and only one of them, namely $(+,-,(0,0),3,1)$, has four supernumerary supersymmetries due to the fact that it has no worldvolume directions along the y coordinates.

The result of this analysis is summarized in the table 3.

Again, we have only included in the table the configurations which preserve some supersymmetries.

3. Supergravity solutions

In this section we are going to develop the formalism needed to obtain supergravity backgrounds for the brane-wave intersections studied in the brane probe approach of the previous section. We will also analyze the degrees of supersymmetry of the different solutions and compare them with the ones corresponding to the brane probe.

The eleven dimensional metrics of the solutions we will be dealing with are warped generalizations of the line element written in eq. (2.1), namely:

$$ds_{11}^2 = h_1 \left(2dx^+dx^- + W\,(dx^+)^2 + (dx^a)^2\right) + h_2\,(d\tilde{x}^{\,\alpha})^2\,, \tag{3.1}$$

where we have distinguished between transverse coordinates parallel to the brane worldvolume (x^a) and those orthogonal to it ($\tilde{x}^{\,\alpha}$). The warp factors h_1 and h_2 are taken to depend on the external coordinates $\tilde{x}^{\,\alpha}$, whereas the profile W can depend on both sets of coordinates. This restricted class of ansatz is similar to the one of standard intersections of branes, that usually exhibit smearing along some relative transverse directions. Of course, a more general ansatz might lead to localized supersymmetric intersections of branes and pp-waves. With some hindsight in mind, we will take h_1 and h_2 to be the same powers of a single harmonic function H that appear in the pure M2 and M5 brane solutions:

$$\begin{aligned} h_1 &= H^{-2/3}\,, \qquad & h_2 &= H^{1/3}\,, \qquad & (\mathrm{M2})\,,\\ h_1 &= H^{-1/3}\,, \qquad & h_2 &= H^{2/3}\,, \qquad & (\mathrm{M5})\,. \end{aligned} \tag{3.2}$$

The four-form field F will be taken as a sum of a wave contribution $F_{\rm wave}$ and a brane contribution. We shall assume that $F_{\rm wave}$ is given by the same expression as in eq. (2.2), i.e.:

$$F_{\rm wave} = dx^+\wedge\Theta\,, \tag{3.3}$$

where Θ is defined in eq. (2.3). For an M2-brane extended along (x^+, x^-, x^a), the contribution to the four-form field strength will be given by the standard "electric" ansatz:

$$F_{M2} = dx^+\wedge dx^-\wedge dx^a\wedge dH^{-1}\,, \tag{3.4}$$

while for the M5-brane we will adopt the following magnetic ansatz:

$$F_{M5} = \tilde{*}dH\,. \tag{3.5}$$

In eq. (3.5) $\tilde{*}$ denotes the Hodge dual with respect to the external coordinates $\tilde{x}^{\,\alpha}$ with the Euclidean metric. The total field strength F must satisfy the Bianchi identity, $dF = 0$, and the field equation:

$$d\,{}^*F = \frac{1}{2}\,F\wedge F\,, \tag{3.6}$$

where $*$ denotes the Hodge dual for the eleven dimensional metric (3.1). These equations for F are enough to fix the precise dependence of H on the external coordinates $\tilde{x}^{\,\alpha}$. Indeed, we will show that, in general, H will only depend on some subset of the $\tilde{x}^{\alpha}$'s and it will be a harmonic function on the other external coordinates. The independence of H on some $\tilde{x}^{\alpha}$'s means that the brane is smeared along those directions and, thus, our M-branes are delocalised objects in transverse space. This remark will be relevant when comparing the number of supersymmetries of the supergravity solutions with those obtained within the brane probe approach.

The metric and gauge field must also satisfy Einstein's equations, which, written in flat coordinates, read:

$$R_{\widehat{P}\,\widehat{Q}} = \frac{1}{12}\,F_{\widehat{P}}^{\ \widehat{N}_1\cdots\widehat{N}_3}\,F_{\widehat{Q}\,\widehat{N}_1\cdots\widehat{N}_3} - \frac{1}{144}\,\eta_{\widehat{P}\,\widehat{Q}}\,F^2\,, \tag{3.7}$$

where $R_{\widehat{P}\,\widehat{Q}}$ is the Ricci tensor. The components of this tensor for a metric of the type (3.1) are written in appendix A. By inspecting eqs. (A.8) and (A.9), one realizes that the profile W only enters the $\widehat{+}\,\widehat{+}$ component of the Ricci tensor. Moreover, one easily concludes that the only contribution to the right-hand side of eq. (3.7) for $P = Q = +$ comes from $F_{\rm wave}$. With the purpose of writing this $\widehat{+}\,\widehat{+}$ Einstein equation in a simpler form, let us choose a basis of one-forms $e^{\widehat{M}}$ as in eq. (A.2) and let us introduce the inverse vierbeins $E^{\ M}_{\widehat{P}}$ by means of the relation $dx^M = E^{\ M}_{\widehat{P}}\,e^{\widehat{P}}$. Then, we can write Θ as:

$$\Theta = \frac{1}{6}\,\theta_{\,\widehat{i}\,\widehat{j}\,\widehat{k}}\;e^{\widehat{i}}\wedge e^{\widehat{j}}\wedge e^{\widehat{k}}\,, \tag{3.8}$$

where:

$$\theta_{\,\widehat{i}\,\widehat{j}\,\widehat{k}} = E^{\ l}_{\widehat{i}}\,E^{\ m}_{\widehat{j}}\,E^{\ n}_{\widehat{k}}\,\theta_{lmn}\,. \tag{3.9}$$

Then, it is straightforward to show that the $\widehat{+}\,\widehat{+}$ Einstein equation is equivalent to the following differential equation for the profile W:

$$\partial_a^2\,W + H^{-1}\,\partial_\alpha^2\,W = -\frac{1}{6}\,\theta_{\,\widehat{i}\,\widehat{j}\,\widehat{k}}\,\theta^{\,\widehat{i}\,\widehat{j}\,\widehat{k}}\,. \tag{3.10}$$

(Compare eqs. (3.10) and (2.4)).

Once H is determined from the equation of motion of the gauge field, eq. (3.10) allows to obtain the profile W. Notice that in the passage from curved to flat components in (3.9), new powers of H are introduced and, thus, the right-hand side of (3.10) does, in general, depend on the $\tilde{x}^{\alpha}$ coordinates. Moreover, it can be verified that the other components of Einstein's equations are satisfied by our ansatz of the metric and F, provided H is harmonic, both for the M2 and M5 cases. Therefore, the only non-trivial information we get from (3.7) is just the profile equation (3.10). The solutions of this equation are, in general, different from the values of W for the pure wave. This fact is a manifestation of the back-reaction exerted on the profile of the wave by the presence of the brane [12].

Let us analyze the behavior of our solutions under supersymmetry. A bosonic configuration of eleven dimensional supergravity is invariant under all supersymmetry transformations which do not change the gravitino. The parameter of such transformation is a spinor η which satisfies the so-called Killing spinor equation:

$$\nabla_M\,\eta = \Omega_M\,\eta\,, \tag{3.11}$$

where ∇_M is the covariant derivative for the metric (3.1) and Ω_M is given in eq. (2.8). Notice that Ω_M is linear in the gauge field. Thus, we can split it as:

$$\Omega_M = \Omega_M^w + \Omega_M^{br}\,, \tag{3.12}$$

where Ω_M^w and Ω_M^{br} are, respectively, the contributions to Ω_M of the wave and brane terms of F. Let us define θ as in eq. (2.9) (notice that now θ can depend on the coordinates $\tilde{x}^\alpha$). Then, it is straightforward to show that, both for an M2 or M5 metric, the different components of Ω_M^w are:

$$\begin{aligned}
\Omega_-^w &= 0\,,\\
\Omega_+^w &= -\frac{1}{12}\,\theta\,\left[\,\Gamma_{\hat{-}}\Gamma_{\hat{+}} + 1\,\right]\,,\\
\Omega_a^w &= \frac{1}{24}\,[\,3\theta\Gamma_{\hat{a}} + \Gamma_{\hat{a}}\,\theta\,]\,\Gamma_{\hat{-}}\,,\\
\Omega_\alpha^w &= \frac{H^{1/2}}{24}\,[\,3\theta\Gamma_{\hat{\alpha}} + \Gamma_{\hat{\alpha}}\,\theta\,]\,\Gamma_{\hat{-}}\,.
\end{aligned} \tag{3.13}$$

Moreover, it is rather convenient to define a new spinor ϵ, which is related to η by means of the expression:

$$\eta = H^{\Delta}\,\epsilon\,, \tag{3.14}$$

where the exponent Δ is:

$$\Delta = \begin{cases} -\dfrac{1}{6}\,, & \text{for a M2-brane}\,,\\[2mm] -\dfrac{1}{12}\,, & \text{for a M5-brane}\,. \end{cases} \tag{3.15}$$

Let us now plug the ansatz (3.14) in the Killing spinor equation (3.11). To compute Ω_M^{br} we use the brane term of the gauge field, written in eqs. (3.4) and (3.5). Moreover, we shall impose to η the corresponding M-brane projection $\Gamma_\kappa\,\eta = \eta$, where Γ_κ is given in eq. (2.16) or (2.18), now extended to the whole spacetime, and not just to the worldvolume. Then, one can verify that Ω_M^{br} drops out and we are left with the following set of differential equations for ϵ:

$$\begin{aligned}
\partial_-\,\epsilon &= 0\,,\\
\partial_+\,\epsilon &= \left[\frac{1}{4}\,\partial_a W\,\Gamma_{\hat{a}\,\hat{-}} + \frac{1}{4}\,H^{-1/2}\partial_\alpha W\,\Gamma_{\hat{\alpha}\,\hat{-}} + \Omega_+^w\right]\epsilon\,,\\
\partial_a\,\epsilon &= \Omega_a^w\,\epsilon\,,\\
\partial_\alpha\,\epsilon &= \Omega_\alpha^w\,\epsilon\,,
\end{aligned} \tag{3.16}$$

which, together with the algebraic condition $\Gamma_\kappa\,\epsilon = \epsilon$, determine the Killing spinor ϵ. Let us first find the solutions of the system (3.16) which correspond to standard spinors ϵ^{st} satisfying the condition $\Gamma_{\hat{-}}\,\epsilon^{st} = 0$. In this case, the previous equations reduce to:

$$\begin{aligned}
\partial_-\,\epsilon^{st} &= \partial_a\,\epsilon^{st} = \partial_\alpha\,\epsilon^{st} = 0\,,\\
\partial_+\,\epsilon^{st} &= -\frac{\theta}{4}\,\epsilon^{st}\,,\\
\Gamma_\kappa\,\epsilon^{st} &= \epsilon^{st}\,,
\end{aligned} \tag{3.17}$$

from which we get the following integrability conditions:

$$\begin{aligned} [\,\Gamma_\kappa\,,\,\theta\,]\,\epsilon^{st} &= 0\,, \\ \partial_\alpha\,\theta\,\epsilon^{st} &= 0\,. \end{aligned} \tag{3.18}$$

If these conditions hold, one finds eight standard spinors of the form:

$$\epsilon^{st} = e^{-x^+\,\theta/4}\,\chi^{st}\,, \qquad \Gamma_\kappa\,\chi^{st} = \chi^{st}\,, \qquad \Gamma_{\hat{-}}\,\chi^{st} = 0\,, \tag{3.19}$$

where χ^{st} is a constant spinor.

For general Killing spinors it is easy to find the compatibility conditions between the equations in (3.16) and the condition $\Gamma_\kappa\,\epsilon = \epsilon$. These conditions are:

$$\begin{aligned} [\,\Gamma_\kappa, \Omega^w_a\,]\,\epsilon &= [\,\Gamma_\kappa, \Omega^w_\alpha\,]\,\epsilon = 0\,, \\ [\,\Gamma_\kappa, \Omega^w_+\,]\,\epsilon &= \frac{1}{2}\,H^{-1/2}\,\partial_\alpha\,W\,\Gamma_{\hat{\alpha}\,\hat{-}}\,\epsilon\,, \end{aligned} \tag{3.20}$$

where we have taken into account that Γ_κ always commutes with $\Gamma_{\hat{a}\,\hat{-}}$ and anticommutes with $\Gamma_{\hat{\alpha}\,\hat{-}}$. Other interesting consistency conditions come from the mutual consistency of eqs. (3.16):

$$\partial_\alpha\,\Omega^w_a\,\epsilon = 0\,. \tag{3.21}$$

Eqs. (3.20) and (3.21) are very useful to discard the possibility of having Killing spinors is some cases. Indeed, let us assume that Γ_κ either commutes or anticommutes with θ, as happened in all the cases studied in the previous section. If $[\Gamma_\kappa, \theta] = 0$ it follows that $[\Gamma_\kappa, \Omega_a] = \{\Gamma_\kappa, \Omega_a\} = 0$, whereas when $\{\Gamma_\kappa, \theta\} = 0$ one has $\{\Gamma_\kappa, \Omega_a\} = [\Gamma_\kappa, \Omega_\alpha] = 0$. By using these results in the first equation in (3.20) and in (3.16), one finds:

$$\begin{aligned} [\,\Gamma_\kappa\,,\,\theta\,] = 0 \;&\Longrightarrow\; \Omega^w_\alpha\,\epsilon = 0 \;\Longrightarrow\; \partial_\alpha\,\epsilon = 0\,, \\ \{\,\Gamma_\kappa\,,\,\theta\,\} = 0 \;&\Longrightarrow\; \Omega^w_a\,\epsilon = 0 \;\Longrightarrow\; \partial_a\,\epsilon = 0\,, \end{aligned} \tag{3.22}$$

which means that some Ω_i's must have a zero mode. This, in some cases, is impossible, which allows to discard the existence of certain classes of Killing spinors. Actually, these conditions, although they are not complete, are restrictive enough, as we will see in the particular examples studied in the next section. Only for those configurations which succeed in passing the test of eqs. (3.20)–(3.22) we will try to integrate directly the system (3.16) and, in these cases, a separation of variables is possible and the solution of (3.16) is easily found.

4. M-branes in the maximally SUSY pp-wave

Let us particularize the general formalism of the previous section to the intersection of M-branes and the maximally supersymmetric pp-wave. In this case the four-form field strength will be of the form:

$$F = \mu\,dx^+ \wedge dy^1 \wedge dy^2 \wedge dy^3 + \text{brane term}\,, \tag{4.1}$$

with μ being a constant. Let us consider a $(+,-,m,n)$ configuration and let us take the worldvolume coordinates to be $y^a = (\,y^1,\ldots,y^m\,)$ and $z^a = (\,z^1,\ldots,z^n\,)$. The external coordinates will be $\tilde{y}^{\,\alpha} = y^{m+\alpha}$ for $\alpha = 1,\ldots,3-m$ and $\tilde{z}^{\,\alpha} = z^{n+\alpha}$ for $\alpha = 1,\ldots,6-n$. The metric will be:

$$ds^2_{11} = h_1\,\big(\,2dx^+dx^- + W\,(dx^+)^2 + (dy^a)^2 + (dz^a)^2\,\big) + h_2\,\big(\,(d\tilde{y}^{\,\alpha})^2 + (d\tilde{z}^{\,\alpha})^2\,\big)\,, \tag{4.2}$$

where h_1 and h_2 are taken as in eq. (3.2) in terms of a function H which must be determined from the gauge field equations. Recall that the profile W can be obtained by integrating eq. (3.10).

We are only interested in solutions which are invariant under some amount of supersymmetry. It is not difficult to characterize these solutions. First of all, notice for this pp-wave the matrix θ is proportional to the matrix I defined in eq. (2.38) and, similarly, the Ω_i's are proportional to the matrices written in eq. (2.39). Since Γ_κ either commutes or anticommutes with I, we are in one of the situations considered in eq. (3.22) and, thus, the Killing spinors ϵ must be a zero mode of some of the Ω_i's. This is not possible for supernumerary spinors and, thus, we conclude that our solutions can only have standard spinors. However, according to the first equation in (3.18), the latter can only exist if $[\Gamma_\kappa, I] = 0$, since otherwise $I\,\chi^{st} = 0$, which cannot be satisfied for $\chi^{st} \neq 0$. Thus, we can restrict ourselves to those configurations for which Γ_κ commutes with I, which were precisely the ones studied in section 2.1. Notice, however, that this condition is not enough, since the second equation in (3.18) implies that $\partial_\alpha\theta = 0$. If these conditions hold, the corresponding solution will have eight standard supersymmetries.

4.1 M2-branes

For a $(+,-,m,n)$ configuration of a M2-brane ($m+n=1$), it is straightforward to compute the matrix θ. One gets:

$$\theta = \mu\,H^{\frac{m-1}{2}}\,\Gamma_{\hat{\tilde{y}}^1\hat{\tilde{y}}^2\hat{\tilde{y}}^3}\,, \tag{4.3}$$

from which we obtain the following equation for the profile W:

$$\partial^2_a\,W + H^{-1}\,\partial^2_{\alpha}\,W = -\mu^2\,H^{m-1}\,. \tag{4.4}$$

4.1.1 $(+,-,1,0)$

We saw in section 2.1 that Γ^{M2}_κ commutes with I only in this $m=1$ case. The complete expression of the four-form field strength is now:

$$F = \mu\,dx^+\wedge dy^1\wedge dy^2\wedge dy^3 + dx^+\wedge dx^-\wedge dy^1\wedge dH^{-1}\,. \tag{4.5}$$

Notice that the Bianchi identity $dF = 0$ is automatically satisfied. Moreover, the wave term in F gives rise to the following component of the Hodge dual field strength:

$${}^*F_{x^+z^1\ldots z^6} = \mu\,H\,. \tag{4.6}$$

Since, $F\wedge F = 0$, the field equation (3.6) reduces to $d^*F = 0$. By inspecting the component (4.6) of ${}^*F$ one arrives at the conclusion that H must depend only on the z coordinates, i.e.:

$$H = H(\,\vec{z}\,)\,. \tag{4.7}$$

Thus our M2-brane is smeared in the (y^2, y^3) directions. The full expression of ${}^*F$ is:

$$ {}^*F = \mu\, H\, dx^+ \wedge dz^1 \wedge \cdots \wedge dz^6 - dy^2 \wedge dy^3 \wedge \tilde{*} dH\,, \tag{4.8} $$

where $\tilde{*}$ denotes now the Hodge dual with respect to the coordinates $z^1 \cdots z^6$ with the Euclidean metric. The equation $d^*F = 0$ for the second term in ${}^*F$ implies that H must be a harmonic function of $z^1 \cdots z^6$. Let us write it as:

$$ H = 1 + \frac{Q}{|\,\vec{z}\,|^4}\,, \tag{4.9} $$

where Q is a constant related to the charge of the M2-brane. To determine completely the metric, let us write the profile equation (4.4) for this $m = 1$ case:

$$ \partial^2_{y^1}\, W + H^{-1}\, \left[\, \partial^2_{\tilde{y}^\alpha} \,+\, \partial^2_{\tilde{z}^\alpha} \,\right]\, W = -\mu^2\,. \tag{4.10} $$

In order to solve this equation, let us represent W as:

$$ W = -\left(\frac{\mu}{3}\right)^2\, \vec{y}^{\,2} - \left(\frac{\mu}{6}\right)^2\, \vec{z}^{\,2} + f(\vec{z})\,, \tag{4.11} $$

where $f(\vec{z})$ is a function to be determined. Notice that the ansatz (4.11) ensures that $f = 0$ for $Q = 0$. By plugging the expression (4.9) for H and our ansatz (4.11) for W in eq. (4.10), one gets that $f(\vec{z})$ satisfies the equation:

$$ \partial^2_{\vec{z}}\, f = -\frac{7}{9}\,\mu^2\, \frac{Q}{|z|^4} \tag{4.12} $$

This type of equation has been solved in general in appendix B. Particularizing to the case of eq. (4.12), we conclude that, up to a harmonic function, f is:

$$ f = \frac{7}{36}\,\mu^2\, Q \frac{1}{|z|^2}\,. \tag{4.13} $$

Then, the full profile for the $(+, -, 1, 0)$ configuration is:

$$ W = -\left(\frac{\mu}{3}\right)^2\, \vec{y}^{\,2} - \left(\frac{\mu}{6}\right)^2\, \vec{z}^{\,2} + \frac{7}{36}\,\mu^2\, Q\, \frac{1}{\vec{z}^{\,2}}\,. \tag{4.14} $$

Notice that, at large distances from the brane, the back-reaction term f is subleading with respect to the one corresponding to the pure pp-wave. This, as we will verify case by case, is a general fact for the solutions we will obtain.

We have already argued in general that M-branes in this pp-wave can only have standard spinors. For this $(+, -, 1, 0)$ embedding $\theta = \mu I$, which commutes with Γ_κ and is independent of the external coordinates. Thus, the two conditions of eq. (3.18) are satisfied and we have eight standard spinors of the form displayed in eq. (3.19).

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4.2 M5 branes

For a $(+,-,m,n)$ M5-brane configuration the wave contribution to F in eq. (4.1) gives rise to the following component of ${}^*F$:

$$ {}^*F_{x^+z^1\cdots z^6} = \mu\, H^{m-1}\,. \tag{4.15} $$

Thus, for $m \neq 1$, the equation $d^*F = 0$ implies that H must be independent of the y coordinates and, therefore:

$$ H = H(\,\tilde{z}^\alpha\,)\,. \tag{4.16} $$

We have seen in section 2.1.2 that Γ_κ commutes with I only for $m = 0, 2$. Then, from now on, we will only consider these two cases, for which eq. (4.16) must hold. Notice that this means that our branes must be smeared along the external y directions. The full ansatz for F will be:

$$ F = \mu\, dx^+ \wedge dy^1 \wedge dy^2 \wedge dy^3 + \tilde{*} dH \wedge dy^{m+1} \wedge \cdots \wedge dy^3\,, \tag{4.17} $$

where again $\tilde{*}$ denotes the Hodge dual with respect to the $\tilde{z}^\alpha$ coordinates with the Euclidean metric. Notice that $F \wedge F = 0$ for $m = 0, 2$. The Bianchi identity is now non-trivial and imposes that H is a harmonic function of the $\tilde{z}$ coordinates:

$$ \partial^2_{\tilde{z}^\alpha}\, H = 0\,. \tag{4.18} $$

Moreover, the full expression for the Hodge dual is now:

$$ \begin{aligned} {}^*F &= \mu\, H^{m-1}\, dx^+ \wedge dz^1 \wedge \cdots \wedge dz^6 \,+ \\ &\quad + dx^+ \wedge dx^- \wedge dy^1 \wedge \cdots \wedge dy^m \wedge dz^1 \wedge \cdots \wedge dz^n \wedge dH^{-1}\,, \end{aligned} \tag{4.19} $$

and the equation of motion is satisfied as a consequence of eq. (4.16).

For these $(+,-,m,n)$ M5-brane configurations the matrix θ is given by:

$$ \theta = \mu\, H^{\frac{m-2}{2}}\, \Gamma_{\hat{y}^1\hat{y}^2\hat{y}^3}\,, \tag{4.20} $$

and, therefore, the profile equation becomes:

$$ \partial^2_a\, W + H^{-1}\, \partial^2_\alpha\, W = -\mu^2\, H^{m-2}\,. \tag{4.21} $$

In what follows we will analyze separately the $m = 0$ and $m = 2$ cases. Notice, however, that only for $m = 2$ the matrix θ in eq. (4.20) is independent of the external coordinates and, thus, only for this case the corresponding supergravity solution is supersymmetric.

4.2.1 $(+,-,2,2)$

According to eq. (4.16), the harmonic function H will only depend on the four external z coordinates $\tilde{z}^\alpha = (\,z^3, \cdots, z^6\,)$. Thus, we can write:

$$ H = 1 + \frac{Q}{|\,\tilde{z}\,|^2}\,. \tag{4.22} $$

Moreover, it follows from eq. (4.21) that the profile equation in this case becomes:

$$\left[\,\partial^2_{y^a} + \partial^2_{z^a}\,\right]\,W + H^{-1}\,\left[\,\partial^2_{\tilde{y}^\alpha} + \partial^2_{\tilde{z}^\alpha}\,\right]\,W = -\mu^2\,. \tag{4.23}$$

Let us try to find a solution to equation (4.23) of the form:

$$W = -\left(\frac{\mu}{3}\right)^2\,\vec{y}^{\,2} - \left(\frac{\mu}{6}\right)^2\,\vec{z}^{\,2} + f(\tilde{z})\,, \tag{4.24}$$

where we have assumed that the unknown function f depends on the same variables as the harmonic function H in eq. (4.22). After substituting the ansatz (4.24) in (4.23), one arrives at the following differential equation for $f(\tilde{z})$:

$$\partial^2_{\tilde{z}^\alpha}\,f = -\frac{4\mu^2}{9}\,\frac{Q}{|\,\tilde{z}\,|^2}\,, \tag{4.25}$$

whose solution can be obtained from the results of appendix B, namely:

$$f = -\frac{1}{9}\,\mu^2 Q\,\log(\,\tilde{z}^{\,2}\,)\,. \tag{4.26}$$

Therefore, the brane contribution to the profile is, in this case, a logarithmic function. Actually, the full profile is given by:

$$W = -\left(\frac{\mu}{3}\right)^2\,\vec{y}^{\,2} - \left(\frac{\mu}{6}\right)^2\,\vec{z}^{\,2} - \frac{1}{9}\,\mu^2 Q\,\log(\,\tilde{z}^{\,2}\,)\,, \tag{4.27}$$

and, as this solution satisfies eq. (3.18), it has eight standard spinors of the form (3.19).

4.2.2 $(+,-,0,4)$

The external z coordinates in this case can be taken as $\tilde{z}^\alpha = (\,z^5, z^6\,)$ and the solution of eq. (4.18) gives a harmonic function which is logarithmic in the $\tilde{z}^\alpha$ coordinates:

$$H = 1 + Q\,\log(\,\tilde{z}^2\,)\,. \tag{4.28}$$

Moreover, the profile equation (4.21) for this case is:

$$\partial^2_{z^a}\,W + H^{-1}\,\left[\,\partial^2_{\tilde{y}^\alpha} + \partial^2_{\tilde{z}^\alpha}\,\right]\,W = -H^{-2}\,\mu^2\,. \tag{4.29}$$

Due to the presence of the logarithm in the expression of H, the solution of (4.29) is not obtainable in terms of elementary functions and we will not try to find it. Notice that now $\theta = \mu H^{-1} I$ (see eq. (4.20)) and, thus, the second equation in (3.18) cannot be satisfied. Therefore, this case is not supersymmetric unless we smear completely the brane by taking H constant which gives rise, after some coordinate redefinition, to the original pp-wave.

5. M-branes in the 24-SUSY pp-wave

Let us split the transverse coordinates x^i as in section 2.2, namely $x^i = (\vec{y}, \vec{z}, x^9)$, where $\vec{y}$ and $\vec{z}$ are vectors with four components. Sometimes it will be useful to differentiate between coordinates parallel and orthogonal to the brane worldvolume. We will use the same conventions as in previous sections, namely, the coordinates along the brane worldvolume will be labeled by a latin index, whereas those transverse to the brane will have greek indices and a tilde. The four-form gauge field strength for the intersection of an M-brane and the pp-wave with 24 supersymmetries will be of the form:

$$F = \mu \, \left[dx^+ \wedge dy^1 \wedge dy^2 \wedge dx^9 + dx^+ \wedge dy^3 \wedge dy^4 \wedge dx^9 \right] + \text{brane term} \, . \tag{5.1}$$

It is rather easy to reach the conclusion that the only configurations which can be supersymmetric are those for which Γ_κ commutes or anticommutes with θ. Indeed, θ is now the sum of two terms $\theta = \theta_1 + \theta_2$, where θ_1 and θ_2 are proportional to a single product of transverse Γ-matrices. Thus θ_1 and θ_2 cannot have zero modes. Let us assume, say, that $[\Gamma_\kappa, \theta_1] = 0$ and $\{\Gamma_\kappa, \theta_2\} = 0$. We will now prove that there are not Killing spinors in this case. First of all, the first condition in (3.18) implies that ϵ^{st} must be a zero mode of θ_2, which is impossible. This excludes the possibility of having standard Killing spinors. On the other hand, the condition $[\Gamma_\kappa, \Omega_a^w]\epsilon = 0$ of eq. (3.20) implies $\theta_2 \Gamma_{\hat{-}}\, \chi = 0$, which cannot be satisfied by supernumerary Killing spinors (for which $\Gamma_{\hat{-}}\, \chi \neq 0$). This reduces our analysis to the configurations listed in tables 2 and 3 for the M2 and M5 branes respectively. In each case we have to study the fulfillment of eqs. (3.18) and (3.22) for standard and supernumerary spinors respectively. At this point it is interesting to recall that the Ω_y matrices do not have supernumerary zero modes which, in most of the embeddings, excludes the possibility of having supernumerary Killing spinors. As in the maximally SUSY case, we will proceed through a case by case study.

5.1 M2-branes

For a M2-brane embedding of the type $(+, -, (m_1, m_2), n, p)$, the θ matrix is:

$$\theta = \mu \, \left[H^{\frac{m_1+p-1}{2}} \, \Gamma_{\hat{y}^1 \hat{y}^2 \hat{x}^9} + H^{\frac{m_2+p-1}{2}} \, \Gamma_{\hat{y}^3 \hat{y}^4 \hat{x}^9} \right] , \tag{5.2}$$

and, therefore, the profile equation becomes:

$$\partial_a^2 \, W + H^{-1} \, \partial_{\alpha}^2 \, W = -\mu^2 \, \left[H^{m_1+p-1} + H^{m_2+p-1} \right] . \tag{5.3}$$

5.1.1 $(+, -, 0, 1, 0)$

This case corresponds to taking $m_1 = m_2 = p = 0$. By computing the Hodge dual of the wave term in (5.1) one can prove that ${}^*F$ has the following components:

$$({}^*F)_{x^+ z^1 \cdots z^4 y^1 y^2} = ({}^*F)_{x^+ z^1 \cdots z^4 y^3 y^4} = \mu \, . \tag{5.4}$$

Notice that no power of H appears on the right-hand side of eq. (5.4). As a consequence the components of (5.4) do not contribute to d^*F and there will be no need of smearing the M2-brane. Without loss of generality we shall extend the M2-brane along the z^1 direction. Accordingly, the brane term of F is given by:

$$F_{M2} = dx^+ \wedge \, dx^- \wedge dz^1 \wedge dH^{-1} \, . \tag{5.5}$$

Then, it follows from eqs. (5.1) and (5.5) that $dF = 0$ automatically and, by computing ${}^*F_{M2}$, one can check that $d^*F = 0$ if H is a harmonic function of all the eight external coordinates $y = (y^1, \cdots, y^4)$, $\tilde{z} = (z^2, z^3, z^4)$ and x^9:

$$H = 1 + \frac{Q}{[\,y^2 + \tilde{z}^2 + (x^9)^2\,]^3}\,. \tag{5.6}$$

Moreover, the profile equation in this case is:

$$\partial_a^2 W + H^{-1}\,\partial_\alpha^2 W = -2\mu^2 H^{-1}\,, \tag{5.7}$$

and is solved by taking:

$$W = -\frac{\mu^2}{4}\,y^2\,. \tag{5.8}$$

Notice that there is no correction with respect to the pure pp-wave term.

Let us now study the supersymmetries of this embedding. Notice, first of all, that $\theta = \mu\, H^{-\frac{1}{2}}\, J$ for this case (see eq. (5.2)) and that $\{\Gamma_\kappa, \theta\} = 0$. Then, in order to satisfy eq. (3.18) we have to require that ϵ^{st} be a zero mode of θ. Thus, the standard Killing spinors are of the form displayed in eq. (3.19) with the extra condition $J\chi^{st} = 0$, which gives four of them. Moreover, from the requirements of (3.22) we conclude that the supernumerary Killing spinors must be annihilated by Ω_{z^1}, which is only possible if they are also annihilated by $J\Gamma_{\underline{-}}$. This, in turn, implies that $\Omega_{z^2}\epsilon = \Omega_{z^3}\epsilon = \Omega_{z^4}\epsilon = 0$. With this information, and the explicit form of the matrices θ and Ω_{y^i}, it is easy to integrate the system (3.16) for supernumerary Killing spinors. The result is just the one written in eqs. (2.55) and (2.56), with the extra condition $\Gamma_\kappa \chi^{sn} = \chi^{sn}$, which gives four supernumerary spinors. Thus, summing up, this system is $8(4+4)$ supersymmetric.

5.1.2 $(+,-,0,0,1)$

The M2-brane in this case is extended along the x^9 direction and, therefore, the brane term in the four-form is:

$$F_{M2} = dx^+ \wedge\; dx^- \wedge dx^9 \wedge dH^{-1}\,. \tag{5.9}$$

As now the wave contribution to F gives rise to the following components of ${}^*F$:

$$({}^*F)_{x^+ z^1 \cdots z^4 y^1 y^2} = ({}^*F)_{x^+ z^1 \cdots z^4 y^3 y^4} = \mu\, H\,, \tag{5.10}$$

then, $d^*F = 0$ if the brane is smeared in the y directions and H is a harmonic function of the z coordinates. Thus:

$$H = 1 + \frac{Q}{z^2}\,. \tag{5.11}$$

Moreover, the profile equation:

$$\partial_a^2 W + H^{-1}\,\partial_\alpha^2 W = -2\mu^2\,, \tag{5.12}$$

is solved by the following function:

$$W = -\frac{\mu^2}{4}\,y^2 - \frac{\mu^2 Q}{2}\,\log(z^2)\,. \tag{5.13}$$

For this embedding $\theta = \mu J$ (see eq. (5.2)) and Γ_κ commutes with θ. Therefore, eq. (3.18) is automatically satisfied. On the other hand, eq. (3.22) implies, in particular, that $\Omega_{y^i}\epsilon = 0$, which is not possible for spinors with $\Gamma_{\hat{-}}\,\epsilon \neq 0$. Thus, there are no supernumerary Killing spinors and this configuration is $8(8+0)$ supersymmetric.

5.2 M5-branes

Let us consider a M5-brane embedding of the type $(+,-,(m_1,m_2),n,p)$. By computing with the metric of this configuration the contribution to ${}^*F$ of the wave term in (5.1), one gets:

$$
\begin{aligned}
({}^*F)_{x^+z^1\cdots z^4y^1y^2} &= \mu\, H^{m_2+p-1}\,,\\
({}^*F)_{x^+z^1\cdots z^4y^3y^4} &= \mu\, H^{m_1+p-1}\,.
\end{aligned}
\tag{5.14}
$$

The study of the powers of H on the right-hand side of eq. (5.14) for the different cases will allow us to determine the precise form of H. Moreover, the matrix θ is now given by:

$$
\theta = \mu\,\Big[H^{\frac{m_1+p-2}{2}}\,\Gamma_{\hat{y}^1\hat{y}^2\hat{x}^9} + H^{\frac{m_2+p-2}{2}}\,\Gamma_{\hat{y}^3\hat{y}^4\hat{x}^9}\Big]\,,
\tag{5.15}
$$

and, therefore, the profile equation takes the form:

$$
\partial_a^2\,W + H^{-1}\,\partial_\alpha^2\,W = -\mu^2\,(\,H^{m_1+p-2} + H^{m_2+p-2}\,)\,.
\tag{5.16}
$$

5.2.1 $(+,-,(2,2),0,0)$

By inspecting (5.14) one readily realizes that in this case H should be independent of x^9 and harmonic on the four z coordinates, i.e.:

$$
H = 1 + \frac{Q}{|z|^2}\,,
\tag{5.17}
$$

while the profile equation (5.16) for $m_1 = m_2 = 2$ and $p = 0$ is solved by:

$$
W = -\frac{\mu^2}{4}\,y^2\,.
\tag{5.18}
$$

Moreover, since now $\theta = \mu J$ and $[\Gamma_\kappa,\theta] = 0$, the conditions (3.18) are trivially satisfied and we have eight standard Killing spinors of the form (3.19). On the other hand, it follows from (3.22) that the Killing spinors must be independent of the external coordinates z^i and x^9. Actually, it is not difficult to demonstrate that this configuration has four supernumerary spinors of the type (2.55), where, in addition to (2.56), χ^{sn} satisfies the condition $\Gamma_\kappa\chi^{sn} = \chi^{sn}$. All together this configuration has $12(8+4)$ supersymmetries. This system has also been studied in ref. [17].

5.2.2 $(+,-,(2,0),2,0)$

The external coordinates in this case are $(\tilde{y}\,,\,\tilde{z}\,,\,x^9) = (\,y^3,y^4,z^3,z^4,x^9\,)$. Since $m_2+p-1 = -1$, H should not depend on $\tilde{y} = (\,y^3,y^4\,)$ and on x^9. Therefore H only depends on $\tilde{z} = (\,z^3,z^4\,)$ in the form:

$$
H = 1 + Q\,\log(\tilde{z}^2)\,.
\tag{5.19}
$$

The profile equation cannot be solved in terms of elementary functions. Actually, since now $\theta = \mu\,[\,\Gamma_{\widehat{y}^1\widehat{y}^2\widehat{x}^9} + H^{-1}\,\Gamma_{\widehat{y}^3\widehat{y}^4\widehat{x}^9}\,]$, the second equation in (3.18) cannot be satisfied and there are no standard spinors. It might be equally verified that supernumerary spinors cannot exist and, thus, this supergravity solution is not supersymmetric (unless we put $Q = 0$, which is the original pp-wave).

5.2.3 $(+, -, (0, 0), 4, 0)$

The external coordinates are now the four y's and x^9. Since $m_1 + p - 1 = m_2 + p - 1 = -1$, the harmonic function should not depend on y and x^9, i.e. it must be a constant. Thus, the brane contribution to F vanishes and we obtain the pure pp-wave solution with a redefinition of μ.

5.2.4 $(+, -, (1, 1), 1, 1)$

In this case $\tilde{y} = (y^2, y^4)$, $\tilde{z} = (z^2, z^3, z^4)$ and, since $m_1 + p - 1 = m_2 + p - 1 = 1$, H is independent of $\tilde{y}$ and equal to:

$$H = 1 + \frac{Q}{|\tilde{z}|}\,. \tag{5.20}$$

Moreover, the profile can be taken as:

$$W = -\frac{\mu^2}{4}\,y^2 - \frac{Q\mu^2}{2}\,|\tilde{z}|\,. \tag{5.21}$$

Concerning supersymmetry, as $\theta = \mu J$ and $[\Gamma_\kappa, \theta] = 0$, the conditions (3.18) are trivially satisfied and we have eight standard spinors. Moreover, eq. (3.22) requires the existence of zero modes of the $\Omega_{\tilde{y}}$ matrices, which cannot occur for supernumerary spinors. Therefore this configuration is $8(8 + 0)$ supersymmetric.

5.2.5 $(+, -, (1, 1), 2, 0)$

Now $\tilde{y} = (y^2, y^4)$, $\tilde{z} = (z^3, z^4)$ and, as $m_1 + p - 1 = m_2 + p - 1 = 0$, H must depend on $\tilde{y}$, $\tilde{z}$ and x^9 as follows:

$$H = 1 + \frac{Q}{[\,\tilde{y}^{\,2} + \tilde{z}^{\,2} + (x^9)^2\,]^{3/2}}\,, \tag{5.22}$$

and the profile W is:

$$W = -\frac{\mu^2}{4}\,y^2 - \frac{Q\mu^2}{2}\,\frac{1}{[\,\tilde{y}^{\,2} + \tilde{z}^{\,2} + (x^9)^2\,]^{1/2}}\,. \tag{5.23}$$

For this embedding $\theta = \mu H^{-\frac{1}{2}} J$ and, since $\{\Gamma_\kappa, \theta\} = 0$, one concludes from (3.18) that there are four standard Killing spinors as those written in (3.19) with $J\chi^{st} = 0$. Moreover, it is straightforward to prove that there are no supernumerary Killing spinors and, thus, this configuration is $4(4 + 0)$ supersymmetric.

5.2.6 $(+,-,(2,0),1,1)$

Now $m_2+p-1=0$ and, thus, by requiring that $d^*F=0$, one concludes that H should depend on $\tilde{y}=(y^3,y^4)$ and $\tilde{z}=(z^2,z^3,z^4)$. However, one can check that $F_{\rm wave}\wedge F_{M5}$ is not zero and, therefore, the condition $d^*F=0$ does not guarantee that the equation of motion of F are satisfied (this does not happen in the cases studied so far). Then, in this case we are not able even to solve the supergravity equations of motion for non-trivial H.

5.2.7 $(+,-,(0,0),3,1)$

In this case, $\tilde{y}=y$, $\tilde{z}=z^4$ and, since $m_1+p-1=m_2+p-1=0$, H will depend on y and z^4:

$$H=1+\frac{Q}{[\,y^{\,2}+(z^4)^2\,]^{3/2}}\,, \tag{5.24}$$

and from the profile equation it follows that one can take:

$$W=-\frac{\mu^2}{4}\,y^{\,2}\,. \tag{5.25}$$

Now $\theta=\mu H^{\frac{1}{2}}J$ and Γ_κ anticommutes with θ. Thus, we will satisfy (3.18) if we require, in addition to the requirements of (3.19), that $J\chi^{st}=0$, which gives four standard spinors. The conditions (3.22) can be satisfied by supernumerary spinors and, actually, one can easily integrate the corresponding system of differential equations (3.16). The result are just the spinors displayed in eq. (2.55), with the additional condition $\Gamma_\kappa\chi^{sn}=\chi^{sn}$, which restricts the number of supernumerary spinors to be one half of those of the pure pp-wave, i.e. four. In conclusion this system is $8(4+4)$ supersymmetric.

6. Summary and conclusions

In this paper we have studied supersymmetric intersections of branes and pp-waves in M-theory. We have first looked at this problem by considering brane probes extended along fixed hyperplanes in the pp-wave background geometry, and by using kappa symmetry to determine the number of supersymmetries preserved by the different embeddings. This analysis leads to a series of algebraic conditions to be satisfied by the Killing spinors of the background. We have performed a case by case analysis and we have determined how many standard and supernumerary spinors satisfy these algebraic conditions for M2 and M5 branes in the pp-wave backgrounds with 32, 24 and 20 supersymmetries.

Furthermore, we have obtained supergravity solutions representing the wave-brane intersection, and we have determined the number of supersymmetries they preserve. The metric of these solutions is a warped version of that of the pp-wave, with a profile which is generically different from that of the pure pp-wave case. Moreover, the fulfillment of the equations of motion of the four-form requires in many cases that the M-branes be delocalised along some directions of the four-form pp-wave flux.

In general, the requirements imposed by supersymmetry in the supergravity analysis are more restrictive that those found in the brane probe approach. Due to the delocalisation of the solution, one expects to make contact with the case of the brane probe outside the origin. This happens in most of the cases, except in some ones in which the supersymmetry is completely lost in the supergravity solution, due to the presence of the harmonic function in some terms of the Killing spinor equation. It is also interesting to point out that there are very few cases in which supernumerary spinors survive at the level of the supergravity analysis. All these embeddings share the distinguishing feature that they present no deformation of the wave profile. This fact points towards the possibility of obtaining these configurations as Penrose limits of non-standard intersections [17].

Our analysis has been systematic but by no means completely exhaustive. In the case of the M5 brane probe, for example, one could try to switch on worldvolume gauge fields that change the kappa symmetry matrix and could make some embeddings supersymmetric. This is actually what happens in the type IIB analysis of ref. [9] and in the $(+,-,2,2)$ configurations in the maximally SUSY eleven dimensional pp-wave [29]. On the other hand, it was claimed in ref. [33] that some broken spacetime supersymmetries in the type IIB theory are restored by using worldsheet symmetries. It would be interesting to find an eleven dimensional analogue of this phenomenon. Another possibility is to consider spherical branes. From the matrix theory approach it is known that there are supersymmetric configurations of this kind, i.e. fuzzy spheres, which can be traced back to the giant gravitons of the $AdS_{4,7}\times S^{7,4}$ space. More generally one could also try to find worldvolume solitons (bions) on the pp-wave background.

On the supergravity side, it would be worth to reconsider those cases for which the brane probe approach predicts the existence of a supersymmetric embedding and, however, we have failed in finding a (supersymmetric) solution of the supergravity equations. In these cases we would have to explore the possibility of modifying our general ansatz (notice, for example, that fully localized intersections of pp-waves and D-branes were constructed in ref. [34]). Finally it would be also interesting to reduce our solutions to ten dimensions and apply them the different string theory dualities. Notice that most of our solutions have isometries which allow this dimensional reduction. In this way one expects to find solutions representing branes in Gödel universes [35].

We expect to report on some of these issues elsewhere.

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A. Ricci tensor for wave-brane intersections

Let us consider a metric in D dimensions of the form:

$$ds^2_D = h_1\left(2dx^+dx^- + W\,(dx^+)^2 + (dx^a)^2\right) + h_2\,(\,d\tilde{x}^{\,\alpha}\,)^2\,, \tag{A.1}$$

where $x^a = (x^1, \cdots, x^{p-1})$ and h_1 and h_2 depend on the coordinates $\tilde{x}^{\,\alpha}$, whereas W is a function of both x^a and $\tilde{x}^{\,\alpha}$. We consider the following basis of one-forms:

$$
\begin{aligned}
e^{\widehat{+}} &= h_1^{1/2}\, dx^+\,, & e^{\widehat{-}} &= h_1^{1/2}\left(dx^- + \frac{W}{2}\, dx^+\right),\\
e^{\widehat{a}} &= h_1^{1/2}\, dx^a\,, & e^{\widehat{\alpha}} &= h_2^{1/2}\, d\tilde{x}^{\,\alpha}\,.
\end{aligned}
\tag{A.2}
$$

In this basis $ds_D^2 = 2e^{\widehat{+}}\, e^{\widehat{-}} + e^{\widehat{a}}\, e^{\widehat{a}} + e^{\widehat{\alpha}}\, e^{\widehat{\alpha}}$. The spin connection is:

$$
\begin{aligned}
\omega^{\widehat{+}\,\widehat{\alpha}} &= \frac{1}{2}\,(\,h_1 h_2\,)^{-1/2}\,\partial_\alpha h_1\, dx^+\,,\\
\omega^{\widehat{-}\,\widehat{\alpha}} &= 1/2\,\,(\,h_1 h_2\,)^{-1/2}\,\,\partial_\alpha h_1\,\left(dx^- + \frac{W}{2}\, dx^+\right) + \frac{1}{2}\,\left(\frac{h_1}{h_2}\right)^{1/2}\,\partial_\alpha\, W\, dx^+\,,\\
\omega^{\widehat{-}\,\widehat{a}} &= \frac{1}{2}\,\partial_a W\, dx^+\,,\\
\omega^{\widehat{a}\,\widehat{\alpha}} &= \frac{1}{2}\,(\,h_1 h_2\,)^{-1/2}\,\partial_\alpha h_1\, dx^a\,,\\
\omega^{\widehat{\alpha}\,\widehat{\beta}} &= \frac{1}{2}\,h_2^{-1}\,\,(\partial_\beta h_2\, d\tilde{x}^{\,\alpha} - \partial_\alpha h_2\, d\tilde{x}^{\,\beta}\,)\,.
\end{aligned}
\tag{A.3}
$$

The light-cone components of the Ricci tensor are:

$$
\begin{aligned}
R_{\widehat{-}\,\widehat{-}} &= 0\,,\\
R_{\widehat{+}\,\widehat{+}} &= -\frac{1}{2}\,h_1^{-1}\,\partial_a^2\, W - \frac{1}{2}\,h_2^{-1}\,\partial_\alpha^2\, W +\\
&\quad + h_2^{-1}\,\left[-\frac{p+1}{4}\,\partial_\alpha\,\log h_1 + \frac{p-D+3}{4}\,\partial_\alpha\,\log h_2\right]\,\partial_\alpha\, W\,,\\
R_{\widehat{+}\,\widehat{-}} &= -\frac{1}{2}\,(\,h_1 h_2\,)^{-1}\,\partial_\alpha^2\, h_1 +\\
&\quad + h_2^{-1}\,\left[\frac{1-p}{4}\,\partial_\alpha\,\log h_1 + \frac{p-D+3}{4}\,\partial_\alpha\,\log h_2\right]\,\partial_\alpha \log h_1\,.
\end{aligned}
\tag{A.4}
$$

The components of the Ricci tensor along the worldvolume of the brane are:

$$
\begin{aligned}
R_{\widehat{a}\,\widehat{b}} = \delta_{ab}\,\Big[&-\frac{1}{2}\,(\,h_1 h_2\,)^{-1}\,\partial_\alpha^2\, h_1 +\\
&+\,h_2^{-1}\left(\frac{1-p}{4}\,\partial_\alpha \log h_1 + \frac{p-D+3}{4}\,\partial_\alpha\,\log h_2\right)\partial_\alpha \log h_1\Big]\,.
\end{aligned}
\tag{A.5}
$$

In order to write the components of the Ricci tensor orthogonal to the brane, let us define:

$$
\varphi = \frac{1}{2}\,(p+1)\,\log h_1 + \frac{1}{2}\,(D-p-3)\,\log h_2\,.
\tag{A.6}
$$

Then:

$$
\begin{aligned}
R_{\widehat{\alpha}\,\widehat{\beta}} = h_2^{-1}\,\Big[&-\partial_\alpha\partial_\beta\,\varphi + \frac{1}{2}\,\partial_\alpha \log h_2\,\partial_\beta\,\varphi + \frac{1}{2}\,\partial_\beta \log h_2\,\partial_\alpha\,\varphi\, -\\
&-\,\frac{p+1}{4}\,\partial_\alpha \log h_1\,\partial_\beta \log h_1 - \frac{D-p-3}{4}\,\partial_\alpha \log h_2\,\partial_\beta \log h_2\, -\\
&-\,\frac{\delta_{\alpha\beta}}{2}\,\partial_\gamma^2\,\log h_2 - \frac{\delta_{\alpha\beta}}{2}\,\partial_\gamma\,\log h_2\,\partial_\gamma\,\varphi\Big]\,.
\end{aligned}
\tag{A.7}
$$

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For a metric of M2 type we put $D = 11$, $p = 2$, $h_1 = H^{-2/3}$ and $h_2 = H^{1/3}$. We get:

$$
\begin{aligned}
R_{\hat{+}\hat{+}} &= -\frac{1}{2}\, H^{2/3}\, \partial_a^2\, W - \frac{1}{2}\, H^{-1/3}\, \partial_\alpha^2\, W\,, \\
R_{\hat{+}\hat{-}} &= -\frac{1}{3}\, H^{-7/3}\, (\,\partial_\alpha\, H\,)^2 + \frac{1}{3}\, H^{-4/3}\, \partial_\alpha^2\, H\,, \\
R_{\hat{a}\,\hat{b}} &= -\frac{1}{3}\, \delta_{ab}\, H^{-7/3}\, (\,\partial_\alpha\, H\,)^2 + \frac{1}{3}\, \delta_{ab}\, H^{-4/3}\, \partial_\alpha^2\, H\,, \\
R_{\hat{\alpha}\,\hat{\beta}} &= -\frac{1}{2}\, H^{-7/3}\, \partial_\alpha H\, \partial_\beta H + \frac{\delta_{\alpha\beta}}{6}\, \Big[\, H^{-7/3}\, (\,\partial_\gamma\, H\,)^2 - H^{-4/3}\, \partial_\gamma^2\, H\,\Big]\,.
\end{aligned}
\tag{A.8}
$$

For a metric of M5 type we put $D = 11$, $p = 5$, $h_1 = H^{-1/3}$ and $h_2 = H^{2/3}$. We obtain:

$$
\begin{aligned}
R_{\hat{+}\hat{+}} &= -\frac{1}{2}\, H^{1/3}\, \partial_a^2\, W - \frac{1}{2}\, H^{-2/3}\, \partial_\alpha^2\, W\,, \\
R_{\hat{+}\hat{-}} &= -\frac{1}{6}\, H^{-8/3}\, (\,\partial_\alpha\, H\,)^2 + \frac{1}{6}\, H^{-5/3}\, \partial_\alpha^2\, H\,, \\
R_{\hat{a}\,\hat{b}} &= -\frac{1}{6}\, \delta_{ab}\, H^{-8/3}\, (\,\partial_\alpha\, H\,)^2 + \frac{1}{6}\, \delta_{ab}\, H^{-5/3}\, \partial_\alpha^2\, H\,, \\
R_{\hat{\alpha}\,\hat{\beta}} &= -\frac{1}{2}\, H^{-8/3}\, \partial_\alpha H\, \partial_\beta H + \frac{\delta_{\alpha\beta}}{3}\, \Big[\, H^{-8/3}\, (\,\partial_\gamma\, H\,)^2 - H^{-5/3}\, \partial_\gamma^2\, H\,\Big]\,.
\end{aligned}
\tag{A.9}
$$

B. Solution of the profile equation

Suppose that $f(\,\vec{x}\,)$ is a function depending on the d-dimensional vector $\vec{x}$ which satisfies the equation:

$$
\nabla_d^2\, f = \frac{C}{|\,\vec{x}\,|^n}\,,
\tag{B.1}
$$

where ∇_d^2 is the laplacian operator in d dimensions and C is a constant. For $d \neq n$ we have the following solution of the above equation:

$$
f(\,\vec{x}\,) = \begin{cases} \dfrac{C}{(d-n)(2-n)}\, |\,\vec{x}\,|^{2-n}\,, & n \neq 2\,, \\ \dfrac{C}{2(d-2)}\, \log(\vec{x}^{\,2})\,, & n = 2\,. \end{cases}
\tag{B.2}
$$

Notice that the general solution of eq. (B.1) can be obtained by adding a d-dimensional harmonic function to the particular solution displayed in eq. (B.2).

C. M-branes in the 20-SUSY pp-wave

In this appendix the general analysis given in (2.34) will be instrumental. Hereafter, and in order not to clutter the notation, all Γ matrices will be flat by default and therefore hats will be omitted everywhere. The easiest way to construct a pp-wave background that leads to an enhancement of 16 to 20 supersymmetries is to set one of the μ's, say μ_4, equal to zero in (2.7). Moreover, we will also take $\mu_1 + \mu_2 + \mu_3 = 0$, which ensures that the wave profile does not depend on y_9. This amounts to a coordinate split $x_i = (y_1, y_1, y_3, y_4, y_5, y_6, z_7, z_8, y_9)$ with $y_i(z_i)$ tangent (perpendicular) to the flux:

$$F_{\text{wave}} = dx^+ \wedge \left(\mu_1\, dy_1 \wedge dy_2 \wedge dy_9 + \mu_2\, dy_3 \wedge dy_4 \wedge dy_9 + \mu_3\, dy_5 \wedge dy_6 \wedge dy_9\right) . \qquad \text{(C.1)}$$

Using now (2.7), we obtain the following undeformed profile:

$$W_0 = -\frac{\mu_1^2}{4}(y_1^2 + y_2^2) - \frac{\mu_2^2}{4}(y_3^2 + y_4^2) - \frac{\mu_3^2}{4}(y_5^2 + y_6^2) .$$

As there are now three groups of y_a coordinates, namely (y_1, y_2), (y_3, y_4) and (y_5, y_6), generic M-brane embeddings will be labeled by $(+, -, (m_1, m_2, m_3), p, q)$ for a brane that extends along coordinates (x^+, x^-), m_1 out of (y_1, y_2), m_2 inside (y_3, y_4) and m_3 along (y_5, y_6), as well as p out of the (z_7, z_8) and q along y_9. Clearly $m_1 + m_2 + m_3 + p + q$ adds up to 1 for an $M2$ and to 4 for an $M5$.

The 4-form flux receives, as before, two contributions $F = F_{\text{wave}} + F_{\text{brane}}$ coming from the wave and the brane respectively. Typically it is the first piece that will signal the smearing of the harmonic profile H along directions perpendicular to the embedding. This piece, as given in (C.1), satisfies the Bianchi identity $dF_{\text{wave}} = 0$ trivially. However from an analysis of the Maxwell's equations $d^*F_{\text{wave}} = 0$ the following conditions are met for a coordinate y_α to be such that $\partial_\alpha H = 0$:

$M2$	$M5$	smeared coordinates
$m_1 + q \neq 0$	$m_1 + q - 1 \neq 0$	y_1, y_2, y_9
$m_2 + q \neq 0$	$m_2 + q - 1 \neq 0$	y_3, y_4, y_9
$m_3 + q \neq 0$	$m_3 + q - 1 \neq 0$	y_5, y_6, y_9

In the next paragraphs we shall investigate case by case all possible embeddings. The full set of equations given in (2.34) will be needed, as the possibility arises now that θ' or θ'' have zero modes, even if they involve part of θ. These cases will typically enforce equality of two of the μ_α (say $\mu_1 = \mu_2$). In this sense it is worthwhile to remind the reader that an equation like $\theta'\chi = 0$ with $\theta' = \mu_1\Gamma_{129} + \mu_2\Gamma_{349} + \mu_3\Gamma_{569}$ and $\mu_1 + \mu_2 + \mu_3 = 0$ is equivalent to a pair of projections, for example $\Gamma_{1234}\chi = \chi$, $\Gamma_{1256}\chi = \chi$, and, therefore, it enforces a 1/4 SUSY projection. However, if $\theta' = \mu_1\Gamma_{129} + \mu_2\Gamma_{349}$, only for $\mu_1 = \mu_2$ there will be zero modes χ and the equation $\theta'\chi = 0$ will be equivalent to the single projection $\Gamma_{1234}\chi = \chi$.

In each case, the analysis will be divided in two parts. First, the brane probe analysis will be carried out. Notice that, in general, the Killing spinors for the supersymmetries of the brane probe embeddings must be a subset of those corresponding to the pure pp-wave configuration. Hence in all cases we must have at least the projection $\Gamma_-\epsilon = 0$ for standard spinors, and $\Gamma_+\chi = \theta\chi = 0$ for supernumeraries.

After that, the sugra analysis can be performed. It involves in addition the following equations:

$$\partial_\alpha\, \theta\, \epsilon = \partial_\alpha \Omega_a \epsilon = \partial_{[\alpha}\Omega_{\beta]}\epsilon = 0 .$$

For standard spinors, only the first one needs to be checked. Also it turns out that the other two in general do not modify the brane-probe analysis with excited scalars. Therefore, as a general rule, the only effect of turning on the back-reaction is to kill standard spinors in some cases.

In what follows we shall list the results of our analysis, both for the M2 and M5 cases. For the sake of simplicity we will only write down the expressions of θ' and θ'' in the warped metric of the wave-brane background. The corresponding values in the brane probe approach can be obtained by setting to one the warp factors. We will also indicate in each case the smearing needed in the supergravity solution, the form of the harmonic function and the profile of the brane-wave background.

C.1 M2 branes

- $\underline{(+,-,(0,0,1),0,0)}$, with the brane extending along y_5, and smeared along y_6, y_9. In this case:

$$\begin{aligned} \theta' &= H^{-1/2}(\mu_1\Gamma_{129} + \mu_2\Gamma_{349})\,; \qquad \theta'' = \mu_3\Gamma_{569}\,, \\ H &= 1 + \frac{Q}{\left(y_1^2+y_2^2+y_3^2+y_4^2+z_7^2+z_8^2\right)^2}\,, \\ W &= W_0 \;+\; \frac{1}{8}\frac{Q\mu_3^2}{y_1^2+y_2^2+y_3^2+y_4^2+z_7^2+z_8^2}\,. \end{aligned} \tag{C.2}$$

  - Brane Probe: Setting $Q=0$ in H first, we see that, only if $\mu_1=\mu_2$, θ' has a chance to have zero modes: $\theta'\chi=0 \Leftrightarrow \Gamma_{1234}\chi=\chi$. In this case we find 4 standard spinors $\epsilon = e^{\frac{x^+}{4}\theta''}\chi$ with $\Gamma_{+-5}\chi=\chi$, $\theta'\chi=\Gamma_-\chi=0$. For supernumeraries we obtain 0, since in this case $\Omega''_\alpha \sim \Gamma_-\Gamma_\alpha\theta''$ has no zero modes. Altogether this gives $4(4+0)$ supersymmetries.
  - Sugra: The additional condition $\partial_\alpha\theta\epsilon=0$ is met iff $\theta'\epsilon=0$. So, we get the same set of projections for standard spinors $\eta = H^{-1/6}\epsilon = H^{-1/6}e^{\frac{x^+}{4}\theta''}\chi$. Since there were already no supernumeraries, we get $4(4+0)$ supersymmetries.

- $\underline{(+,-,(0,0,0),1,0)}$. The brane extends along the z_7 coordinate. There is no smearing and the profile exhibits no deformation:

$$\begin{aligned} \theta' &= H^{-1/2}(\mu_1\Gamma_{129} + \mu_2\Gamma_{349} + \mu_3\Gamma_{569})\,; \qquad \theta''=0\,, \\ H &= 1 + \frac{Q}{\left((\vec{y})^2+\tilde{z}_8^2\right)^3}\,, \\ W &= W_0\,. \end{aligned} \tag{C.3}$$

  - Brane Probe: Setting $Q=0$ in H, we get 2 standard (constant) spinors $\epsilon=\chi$, with $\theta'\chi=\Gamma_-\chi=0$ and $\Gamma_{+-7}\chi=\chi$. Also there are 2 supernumerary spinors $\epsilon = e^{x^\alpha\Omega_\alpha}\chi$ with $\Omega_a\chi=\Omega_7\chi=0 \Leftrightarrow \theta\chi=0$, as well as $\Gamma_+\chi=0$ and $\Gamma_{+-7}\chi=\chi$. We find altogether $4(2+2)$ supersymmetries.

- Sugra: Here also, when $Q \neq 0$, the constraint $\partial_\alpha \theta \epsilon = 0$ is fulfilled if $\theta' \epsilon = 0$. Therefore, the same number of standard spinors as in the brane probe approach occur and $\eta = H^{-1/6}\epsilon = H^{-1/6}\chi$. Also, from (3.13) we see that $\partial_\beta \Omega_\alpha = 0$, and $\partial_\alpha \Omega_a \epsilon \sim \theta' \epsilon = 0$. Thus, two supernumerary spinors also survive: $\eta = H^{-1/6}\epsilon = H^{-1/6} e^{x^\alpha \Omega_\alpha}\chi$. Then, we find $4(2+2)$ supersymmetries.

- $\underline{(+,-,(0,0,0),0,1)}$. The brane extends along y_9 and is smeared along all $y_1, ..., y_6$. Now:

$$\begin{aligned}
\theta' &= 0\,; \qquad \theta'' = \mu_1 \Gamma_{129} + \mu_2 \Gamma_{349} + \mu_3 \Gamma_{569}\,, \\
H &= 1 + Q \log(\,\vec{z}^{\,2}\,)\,, \\
W &= W_0 - \frac{Q(\mu_1^2 + \mu_2^2 + \mu_3^2)}{4}\,\vec{z}^{\,2}\,\left(\log(\,\vec{z}^{\,2}\,) - 1\right)\,.
\end{aligned} \tag{C.4}$$

  - Brane Probe: There are 8 standard spinors $\epsilon = e^{\frac{x^+}{4}\theta''}\chi$ with $\Gamma_{+-9}\chi = \chi$, $\Gamma_-\chi = 0$. We have 2 supernumeraries without scalars $\epsilon = e^{x^9\Omega_9}\chi$, $\Gamma_{+-9}\chi = \chi$, $\Gamma_+\chi = \theta\chi = 0$. With scalars, the condition $\Omega''_\alpha\chi = 0$ is impossible for $\alpha = 1, \ldots, 6$ and, hence, there are no spinors. In summary we obtain $10(8+2)$ supersymmetries without scalars and $8(8+0)$ with scalars.
  - Sugra: If $Q \neq 0$, still $\partial_\alpha\theta = 0$, so the brane-probe analysis is not modified for standard spinors. For supernumeraries, $\partial_\beta\Omega_\alpha \neq 0$ (see (3.13)), hence one must have $\Omega_\alpha\epsilon = 0$, which is again impossible. Therefore we find $8(8+0)$ supersymmetries.

C.2 M5 branes

- $\underline{(+,-,(2,2,0),0,0)}$. The brane extends along $y_1, \ldots, y_4$ and is smeared along y_5, y_6 and y_9. In this case:

$$\begin{aligned}
\theta' &= 0\,; \qquad \theta'' = \mu_1 \Gamma_{129} + \mu_2 \Gamma_{349} + H^{-1}\mu_3 \Gamma_{569}\,, \\
H &= 1 + Q \log(\,\vec{z}^{\,2}\,)\,.
\end{aligned}$$

  - Brane Probe: Setting $Q = 0$ yields 8 standard spinors $\epsilon = e^{\frac{x^+}{4}\theta''}\chi$ with $\Gamma_{+-1234}\chi = \chi$, $\Gamma_-\chi = 0$. Also we get 2 supernumerary spinors without scalars $\epsilon = e^{x^a\Omega_a}\chi$; $\Gamma_{+-1234}\chi = \chi$, $\Gamma_+\chi = \theta\chi = 0$. With scalars, $\Omega''_\alpha\chi = 0$ is impossible for $\alpha = y_5, y_6$, therefore there are no supernumerary spinors in this case. In summary, we obtain $10(8+2)$ and $8(8+0)$ supersymmetries.
  - Sugra: Now, with $Q \neq 0$ the profile W is difficult to solve for. Moreover, we have the additional integrability condition $\partial_\alpha\theta\epsilon = 0$, which is impossible to fulfill because $\partial_\alpha\theta \sim \Gamma_{569}$. Therefore, we do not have any supersymmetry in this case.

- $\underline{(+,-,(1,1,2),0,0)}$. The brane extends along y_1, y_3, y_5 and y_6 and is smeared along y_9. One has:

$$\theta' = H^{-1/2}(\mu_1\Gamma_{129} + \mu_2\Gamma_{349})\,; \qquad \theta'' = \mu_3\Gamma_{569}\,,$$

$$
\begin{aligned}
H &= 1 + \frac{Q}{y_2^2 + y_4^2 + \vec{z}^{\,2}}\,, \\
W &= W_0 + \frac{Q(\mu_1^2+\mu_2^2)}{8} \log(\, y_1^2 + y_2^2 + \vec{z}^{\,2}\,)\,.
\end{aligned}
$$

- Brane-Probe: Setting $Q = 0$, only for $\mu_1 = \mu_2$, four standard spinors exist, $\epsilon = e^{\frac{x^+}{4}\theta''}\chi$, with $\Gamma_{+-1356}\chi = \chi$ and $\theta'\chi = \Gamma_-\chi = 0$. For supernumerary spinors without scalars we must impose $\Omega'_a\chi = 0$. This can have a solution for $a = 5, 6$ if $\mu_1 = \mu_2$, but not for $a = 1, 3$. Therefore we find no supernumerary spinors.
- Sugra: For $Q \neq 0$, the supergravity analysis coincides with the brane-probe analysis because $\partial_\alpha\theta\chi \sim \theta'\chi = 0$ is one of the defining conditions of the standard spinors and, thus, there are no supernumeraries. In all cases we find $4(4 + 0)$ supersymmetries.

• $\underline{(+,-,(1,1,1),1,0)}$. The brane extends along y_1, y_3, y_5 and z_7 with no smearing and:

$$
\begin{aligned}
\theta' &= H^{-1/2}(\mu_1\Gamma_{129} + \mu_2\Gamma_{349} + \mu_3\Gamma_{569})\,; \qquad \theta'' = 0\,, \\
H &= 1 + \frac{Q}{(y_2^2 + y_4^2 + y_6^2 + z_8^2 + y_9^2)^{3/2}}\,, \\
W &= W_0 - \frac{1}{8}\frac{Q(\mu_1^2+\mu_2^2+\mu_3^2)}{(y_2^2 + y_4^2 + y_6^2 + z_8^2 + y_9^2)^{1/2}}\,.
\end{aligned}
$$

- Brane Probe: with $Q = 0$ we find 2 standard spinors $\epsilon = \chi$ ($\Gamma_{+-1357}\chi = \chi, \Gamma_-\chi = \theta'\chi = 0$). Also we find 0 supernumerary spinors because $\Omega'_a\chi = 0$ for $a = z_7$ implies $\theta'\chi = 0$, but then for $a = 1, 3, 5$ there is no solution.
- Sugra: With $Q \neq 0$, the same projections as in the supergravity analysis are obtained, namely $\partial_\alpha\theta\epsilon = 0 \Leftrightarrow \theta' = 0$. Altogether we have 2(2+0) supersymmetries in all cases.

• $\underline{(+,-,(1,1,1),0,1)}$. Now, contrarily to the previous case, smearing occurs along all the y_α directions. Therefore only z_7, z_8 are transverse and we find:

$$
\begin{aligned}
\theta' &= 0\,; \qquad \theta'' = \mu_1\Gamma_{129} + \mu_2\Gamma_{349} + \mu_3\Gamma_{569}\,, \\
H &= 1 + Q\log(z_7^2 + z_8^2)\,, \\
W &= W_0 + \frac{Q}{8}(\mu_1^2+\mu_2^2+\mu_3^2)\ \vec{z}^{\,2}\ \log(\,\vec{z}^{\,2} - 1)\,.
\end{aligned}
$$

- Brane Probe: Putting $Q = 0$ in H, 8 standard spinors are obtained: $\epsilon = e^{\frac{x^+}{4}\theta''}\chi$ ($\Gamma_{+-1359}\chi = \chi$, $\Gamma_-\chi = 0$). Without scalars, we find 2 supernumerary spinors $\epsilon = e^{x^a\Omega_a}\chi$ ($\Gamma_{+-1359}\chi = \chi, \Gamma_+\chi = \theta\chi = 0$). When scalars are excited, $\Omega''_\alpha\chi = 0$ is impossible for $\alpha = 2, 4, 6$. Altogether for this configuration we get $10(8 + 2)$ and $8(8 + 0)$ supersymmetries.
- Sugra: If $Q \neq 0$ no change occurs since θ is independent of H, and there are no supernumerary spinors. So, in this case one also gets $8(8 + 0)$ supersymmetries.

- $\underline{(+,-,(2,1,0),1,0)}$. The harmonic function can only depend on y_3 and z_8, all other directions being either world-volume or smeared. Thus:

$$\begin{aligned} \theta' &= H^{-1/2}\mu_2\Gamma_{349}\,; \qquad \theta'' = \mu_1\Gamma_{129} + H^{-1}\mu_3\Gamma_{569}\,, \\ H &= 1 + Q\log(y_3^2 + z_8^2)\,. \end{aligned}$$

The profile is difficult to solve for. In any case the embedding is not supersymmetric, since θ' in this case has no zero modes.

- $\underline{(+,-,(2,1,0),0,1)}$. In this case:

$$\theta' = H^{1/2}\mu_1\Gamma_{129} + H^{-1/2}\mu_3\Gamma_{569}\,; \qquad \theta'' = \mu_2\Gamma_{349}\,.$$

  - Brane Probe: When $H = 1$ and $\mu_1 = \mu_2$ there are 4 standard spinors $\epsilon = e^{\frac{x^+}{4}\theta''}\chi$ ($\Gamma_{+-1239}\chi = \chi, \Gamma_-\chi = \theta'\chi = 0$). Concerning supernumerary spinors, they must satisfy $\Omega''_\alpha\chi = 0$. For $\alpha = 7, 8$ this is tantamount to $\theta''\chi = 0$, which is impossible, since θ'' has no zero modes. Hence no supernumerary spinors survive and we have $4(4+0)$ supersymmetries.
  - Sugra: From our general rule, the brane should be smeared in the y_4 coordinate. However $F_{\rm wave} \wedge F_{\rm brane}$ is not zero unless the brane is completely smeared and H is constant. Thus, in this case the only supergravity solution we find is the original pp-wave.

- $\underline{(+,-,(2,0,0),2,0)}$. There is smearing along all y_α coordinates. And since z_a are also internal, there is no external volume to the brane, and it dissolves completely, reverting to the original *pp*-wave.

- $\underline{(+,-,(2,0,0),1,1)}$. For this configuration:

$$\theta' = H^{1/2}\mu_1\Gamma_{129} + H^{-1/2}(\mu_2\Gamma_{349} + \mu_3\Gamma_{569})\,; \qquad \theta'' = 0\,.$$

  - Brane Probe: When $H = 1$ we find 2 standard spinors $\epsilon = \chi$ ($\Gamma_{+-1279}\chi = \chi$, and $\Gamma_-\chi = \theta'\chi = 0$) and 0 supernumerary spinors, since $\Omega'_a\chi = 0$ is not possible with $a = 1, 2$. Thus this system is $2(2+0)$ supersymmetric.
  - Sugra: In this case $d^*F = 0$ without any smearing. However, $F_{\rm wave} \wedge F_{\rm brane}$ is zero only when H is constant, which corresponds to the pure pp-wave.

- $\underline{(+,-,(1,1,0),2,0)}$. The brane extends along y_1, y_3, z_7, z_8 and is smeared along y_5, y_6, y_9. Now:

$$\begin{aligned} \theta' &= H^{-1/2}(\mu_1\Gamma_{129} + \mu_2\Gamma_{349})\,; \qquad \theta'' = H^{-1}\mu_3\Gamma_{569}\,, \\ H &= 1 + Q\log(\,y_2^2 + y_4^2\,)\,. \end{aligned}$$

  - Brane Probe: with $Q = 0$, only for $\mu_1 = \mu_2$ there are 4 standard spinors, $\epsilon = e^{\frac{x^+}{4}\theta''}\chi$ ($\Gamma_{+-1378}\chi = \chi, \Gamma_-\chi = \theta'\chi = 0$), and 0 supernumeraries, because θ'' has no zero modes: i.e. this configuration is $4(4+0)$ supersymmetric.

- Sugra: with $Q \neq 0$ all spinors are lost since $\partial_\alpha \theta \epsilon = 0$ has no solution.

- $\underline{(+,-,(1,1,0),1,1)}$. The brane extends along y_1, y_3, z_7, y_9 and is smeared along y_2, y_4. Thus:

$$\begin{aligned} \theta' &= H^{-1} \mu_3 \Gamma_{569} \, ; \qquad \theta'' = \mu_1 \Gamma_{129} + \mu_2 \Gamma_{349} \, , \\ H &= 1 + \frac{Q}{(y_5^2 + y_6^2 + z_8^2)^{1/2}} \, . \end{aligned}$$

No spinors in any case, since θ' has no zero modes. Profile W seems difficult to solve for.

- $\underline{(+,-,(1,0,0),2,1)}$. The brane covers y_1, z_7, z_8, y_9 and is smeared along y_2. Therefore:

$$\begin{aligned} \theta' &= H^{-1/2} (\mu_2 \Gamma_{349} + \mu_3 \Gamma_{569}) \, ; \qquad \theta'' = \mu_1 \Gamma_{129} \, , \\ H &= 1 + \frac{Q}{y_3^2 + y_4^2 + y_5^2 + y_6^2} \, , \\ W &= W_0 + \frac{Q \mu_1^2}{8} \log(y_3^2 + y_4^2 + y_5^2 + y_6^2) \, . \end{aligned}$$

- Brane Probe: When $Q = 0$, only for $\mu_1 = \mu_2$ there are 4 spinors, $\epsilon = e^{\frac{x^+}{4} \theta''} \chi$, with $\Gamma_{+-1789} \chi = \chi$ and $\Gamma_- \chi = \theta' \chi = 0$. No supernumeraries appear because θ'' has no zero modes.
- Sugra: For $Q \neq 0$, the condition $\partial_\alpha \theta \chi = 0$ is fulfilled with $\theta' \chi = 0$. So, the analysis goes through, and we have 4 standard spinors $\eta = H^{-1/12} \epsilon = H^{-1/12} e^{\frac{x^+}{4} \theta''} \chi$, with the same χ as before. In all cases we have $4(4+0)$ supersymmetries.

The previous analysis is summarized in the table 4, where we have included only those cases which preserve some supersymmetry. The asterisk distinguishes those configurations for which $\mu_1 = \mu_2$ has to be enforced in order to have some supersymmetry.

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$M2$	brane probe without scalars	brane probe with scalars	sugra analysis
$(+,-,(0,0,1),0,0)^*$	4(4+0)	4(4+0)	4(4+0)
$(+,-,(0,0,0),1,0)$	4(2+2)	4(2+2)	4(2+2)
$(+,-,(0,0,0),0,1)$	10(8+2)	8(8+0)	8(8+0)
$M5$			
$(+,-,(2,2,0),0,0)$	10(8+2)	8(8+0)	0(0+0)
$(+,-,(1,1,2),0,0)^*$	4(4+0)	4(4+0)	4(4+0)
$(+,-,(1,1,1),1,0)$	2(2+0)	2(2+0)	2(2+0)
$(+,-,(1,1,1),0,1)$	10(8+2)	8(8+0)	8(8+0)
$(+,-,(2,1,0),0,1)^*$	4(4+0)	4(4+0)	-
$(+,-,(2,0,0),1,1)$	2(2+0)	2(2+0)	-
$(+,-,(1,1,0),2,0)^*$	4(4+0)	4(4+0)	0(0+0)
$(+,-,(1,0,0),2,1)^*$	4(4+0)	4(4+0)	4(4+0)

Table 4: Supersymmetric intersections of M2 and M5 branes with the 5/8 SUSY pp-wave background.

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